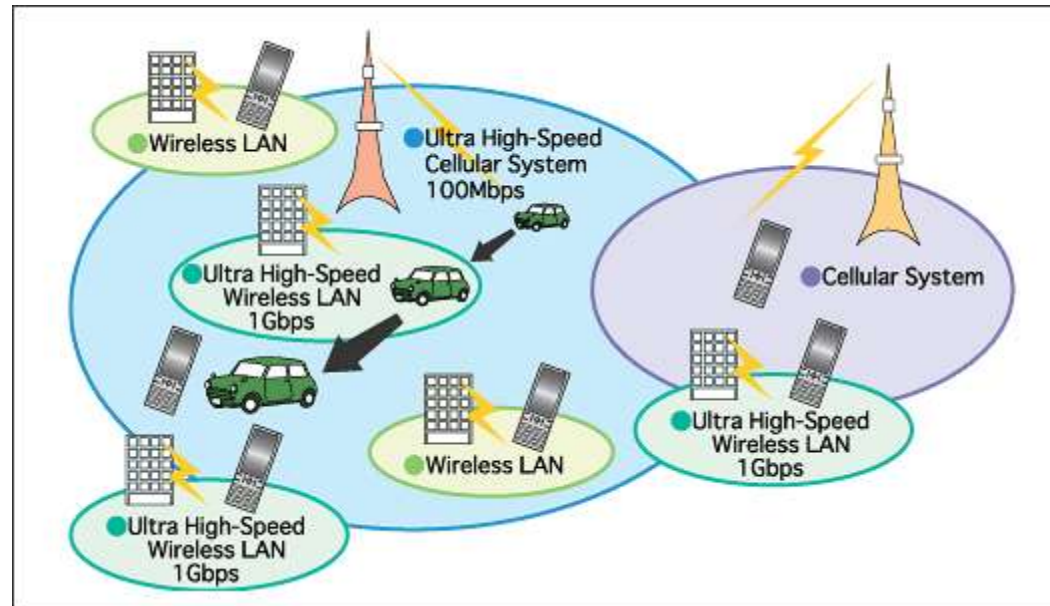


# Communications Systems



- 1 Analogue modulation:* time domain (waveforms), frequency domain (spectra), amplitude modulation (am), frequency modulation (fm), phase modulation (pm)
- 2 Digital modulation:* waveforms and spectra, Frequency Shift Keying (FSK), Binary Phase Shift Keying (BPSK) [including Gaussian Minimum Shift Keying (GMSK)], Quadrature Phase Shift Keying (QPSK) [including  $\pi/4$ QPSK]
- 3 Error coding:* General principles of block, convolutional, parity, interleaving
- 4 Compression:* Regular Pulse Excitation – Linear Predictive Coding – Long Term Prediction (RPE-LPC-LTP)

# Overview

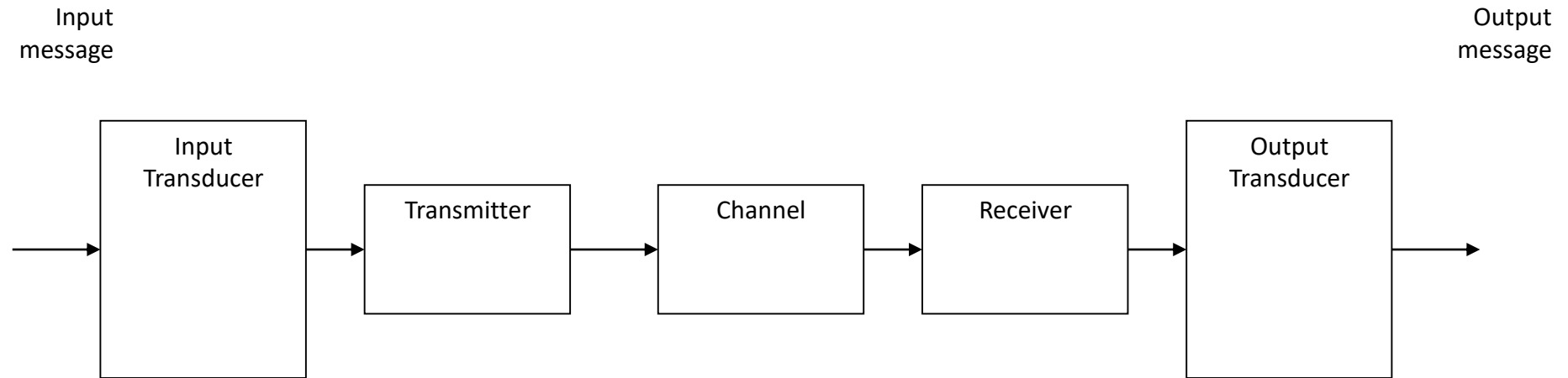
**Communication** is the transfer of information from one place to another.

*This should be done*

- as **efficiently** as possible
- with as much **fidelity/reliability** as possible
- as **securely** as possible

**Communication System:** Components/subsystems act together to accomplish information transfer/exchange.

# Elements of a Communication System



**Input Transducer:** The message produced by a source must be converted by a transducer to a form suitable for the particular type of communication system.

*Example: In electrical communications, speech waves are **converted** by a microphone to voltage variation.*

**Transmitter:** The transmitter processes the input signal to produce a signal suits to the characteristics of the transmission channel.

*Signal **processing** for transmission almost always involves **modulation** and may also include **coding**. In addition to modulation, other functions performed by the transmitter are **amplification**, **filtering** and coupling the modulated signal to the channel.*

**Channel:** The channel can have different forms: The atmosphere (or free space), coaxial cable, fiber optic, waveguide, etc.

*The signal undergoes some amount of degradation from noise, interference and distortion*

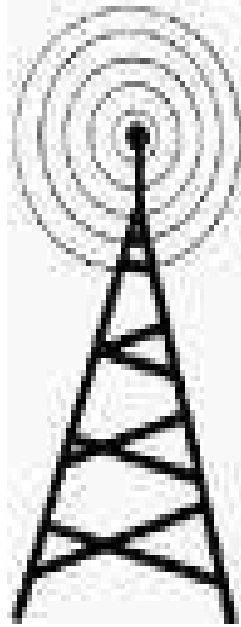
**Receiver:** The receiver's function is to extract the desired signal from the received signal at the channel output and to convert it to a form suitable for the output transducer.

*Other functions performed by the receiver: amplification (the received signal may be extremely weak), demodulation and filtering.*

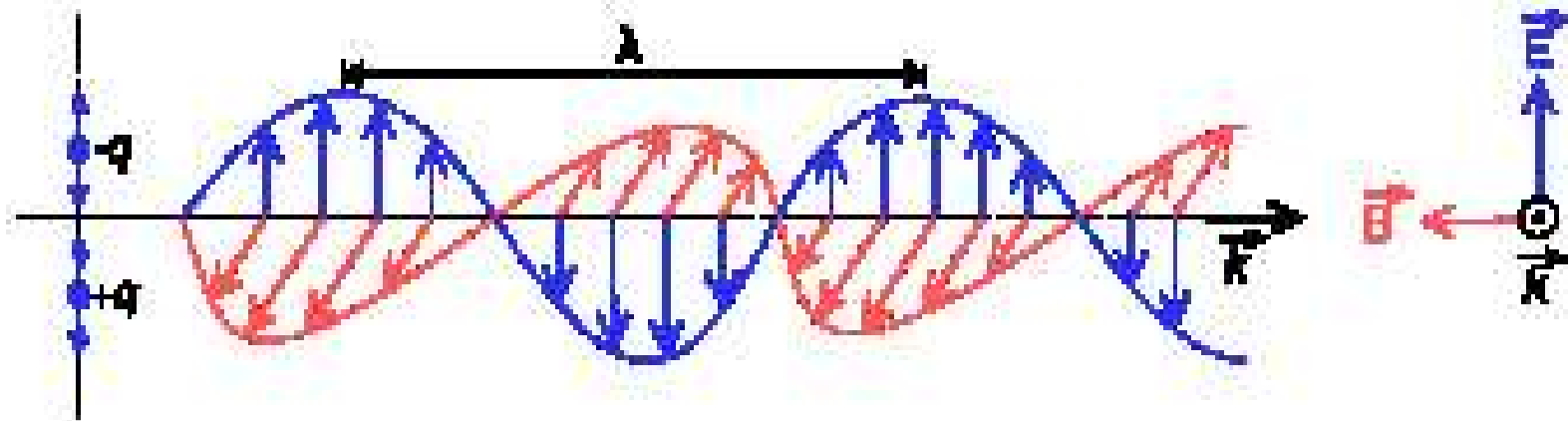
**Output Transducer:** Converts the electric signal at its input into the form desired by the system user.

*Example: Loudspeaker, personal computer (PC), tape recorders.*

To be transmitted, **Information (Data)** must be transformed to electromagnetic signals.



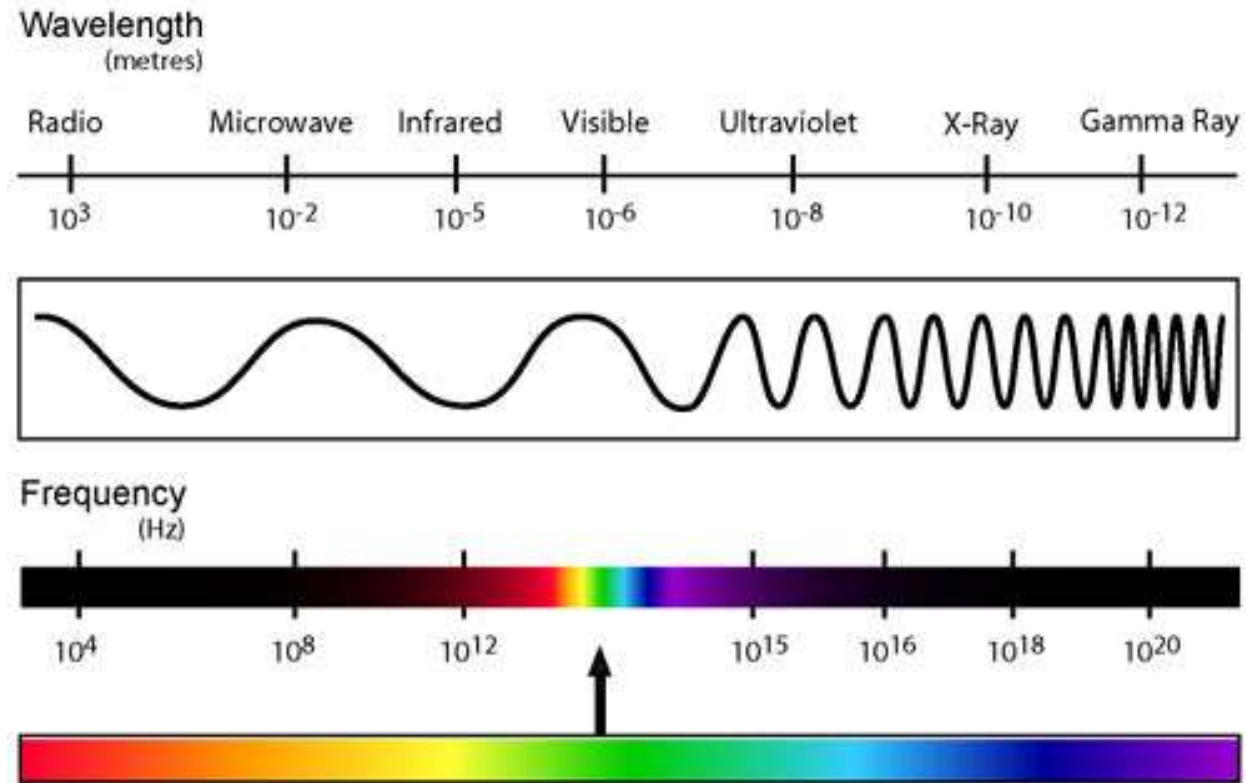
# Electromagnetic Waves





# Electromagnetic Waves

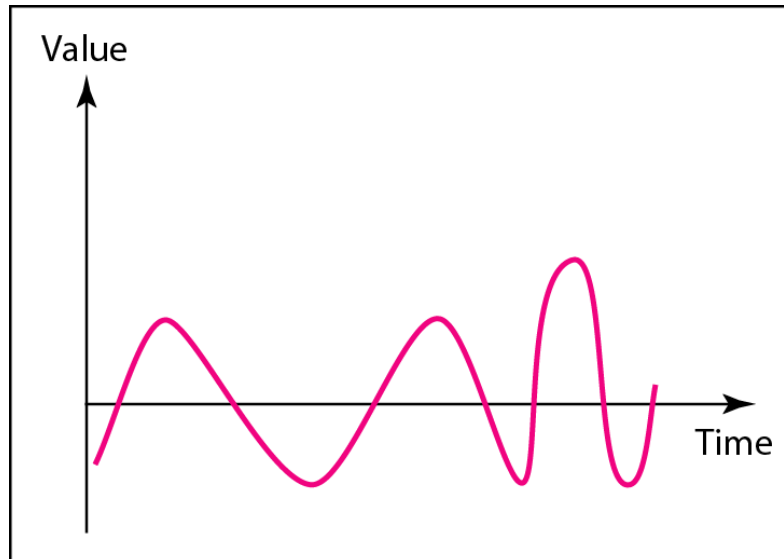
THE ELECTRO MAGNETIC SPECTRUM



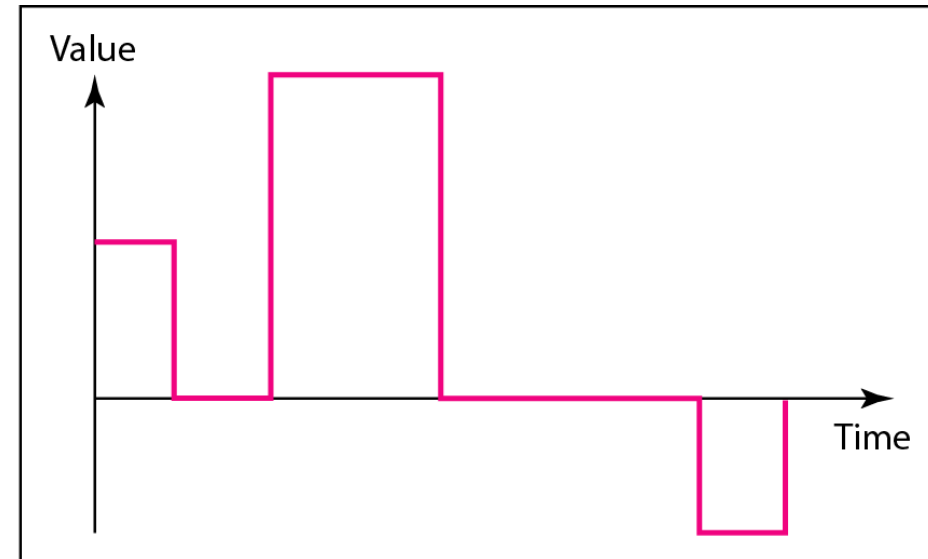
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Figure *Comparison of analog and digital signals*

---



a. Analog signal

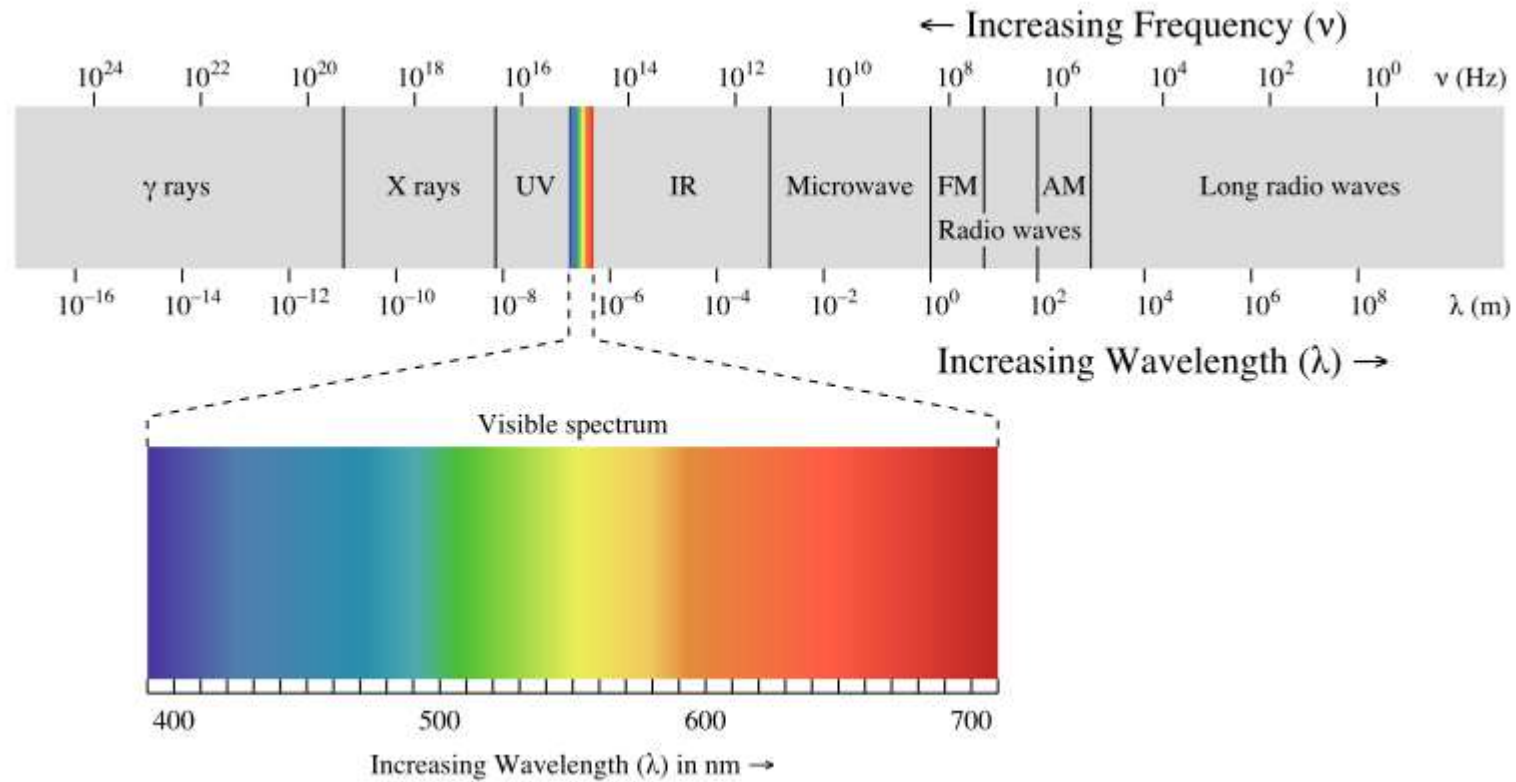


b. Digital signal

---

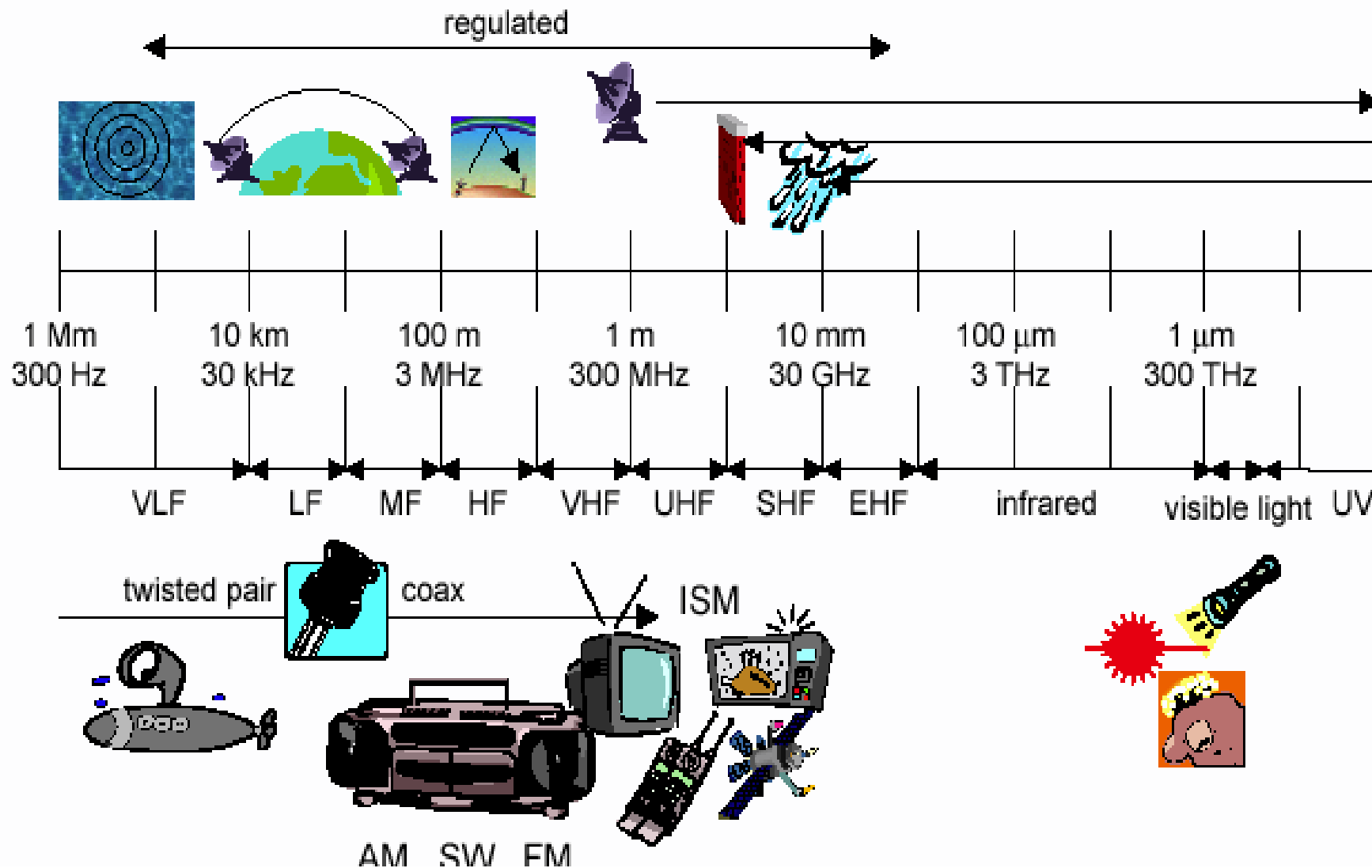


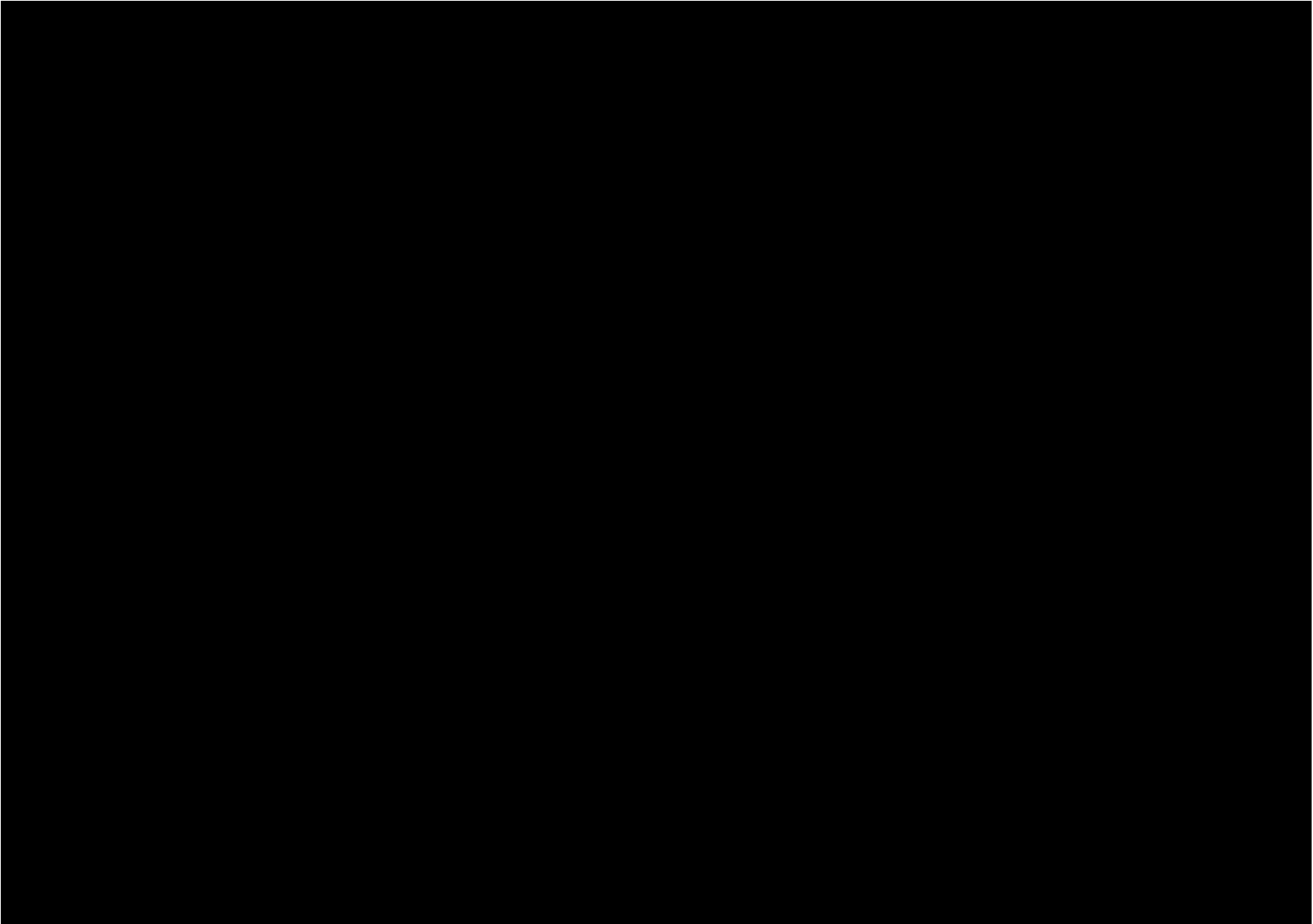
# Electromagnetic Spectrum



[http://www.edumedia-sciences.com/a185\\_l2-transverse-electromagnetic-wave.html](http://www.edumedia-sciences.com/a185_l2-transverse-electromagnetic-wave.html)

# Electromagnetic Spectrum

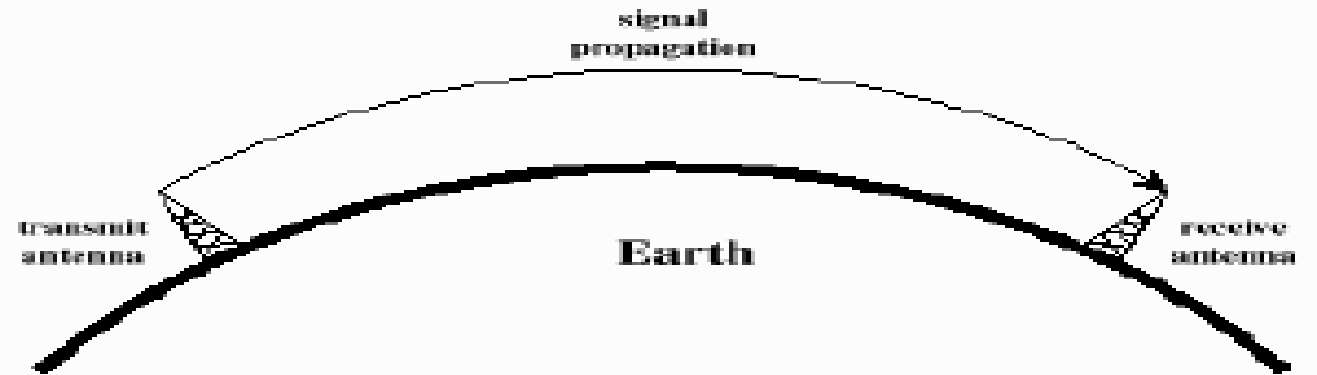




# 1.6 Radio Wave Propagation Modes

## 1 Ground Wave Propagation

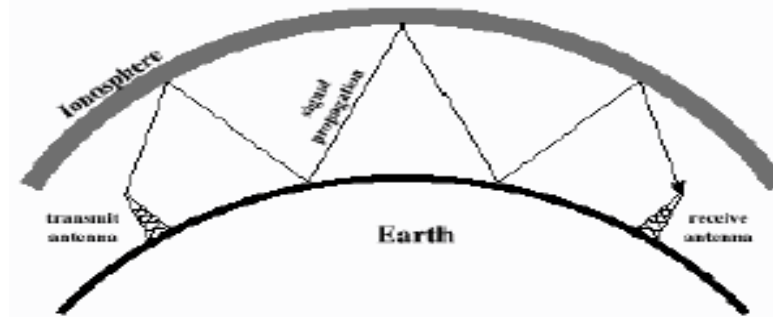
Follows contour of the earth  
*Frequencies up to 2 MHz*



## 2 Sky Wave Propagation

Signal reflected from ionized layer of atmosphere. Signal can travel a number of hops, back and forth

*Examples SW radio*



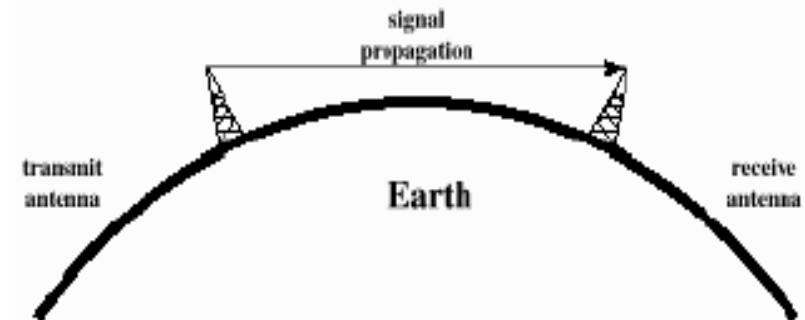
## 3 Line-of-Sight Propagation

Transmitting and receiving antennas must be within line of sight

*example*

*Satellite communication*

*Ground communication*





# ANALOG AND DIGITAL

*Data (Information) can be **analog** or **digital**. The term **analog data** refers to information that is continuous; **digital data** refers to information that has discrete states. Analog data take on continuous values. Digital data take on discrete values.*

---

Data can be analog or digital.

Analog data are continuous and take continuous values.

Digital data have discrete states and take discrete values.

---

Signals can be analog or digital.  
Analog signals can have an infinite number  
of values in a range; digital signals can  
have only a limited  
number of values.

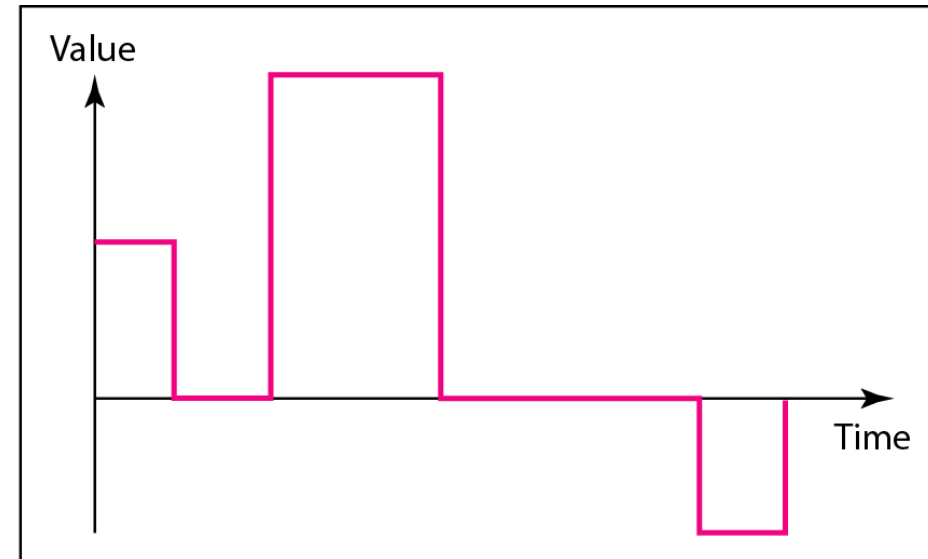
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Figure *Comparison of analog and digital signals*

---



a. Analog signal



b. Digital signal

---

# Lecture 4

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Frequency is the rate of change with respect to time.

Change in a short span of time means high frequency.

Change over a long span of time means low frequency.

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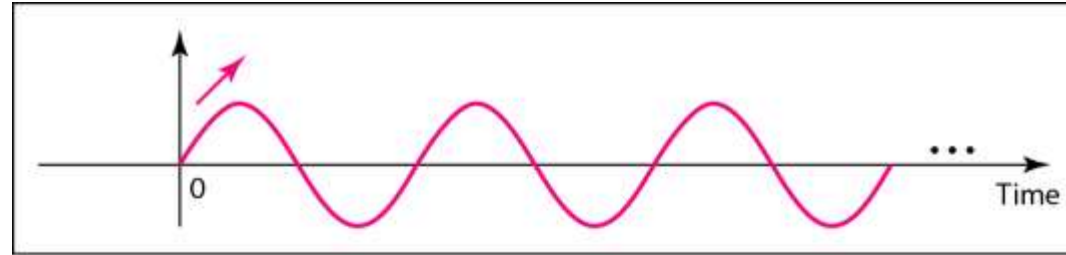
If a signal does not change at all, its  
frequency is zero.

If a signal changes instantaneously, its  
frequency is infinite.

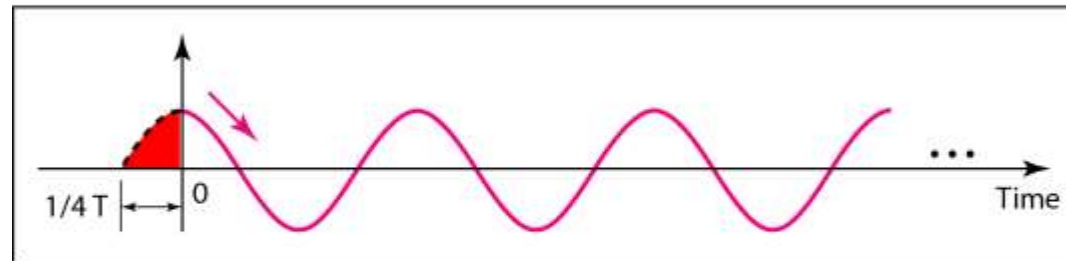
Phase describes the position of the waveform relative to time 0.



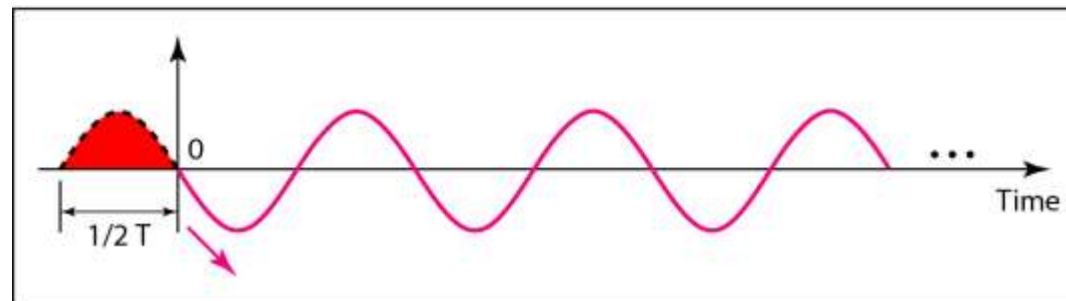
Figure *Three sine waves with the same amplitude and frequency, but different phases*



a. 0 degrees



b. 90 degrees



c. 180 degrees



## *Example*

*A sine wave is offset 1/6 cycle with respect to time 0.  
What is its phase in degrees and radians?*

### *Solution*

*We know that 1 complete cycle is 360°. Therefore, 1/6  
cycle is*

$$\frac{1}{6} \times 360 = 60^\circ = 60 \times \frac{2\pi}{360} \text{ rad} = \frac{\pi}{3} \text{ rad} = 1.046 \text{ rad}$$

Figure *Wavelength and period*

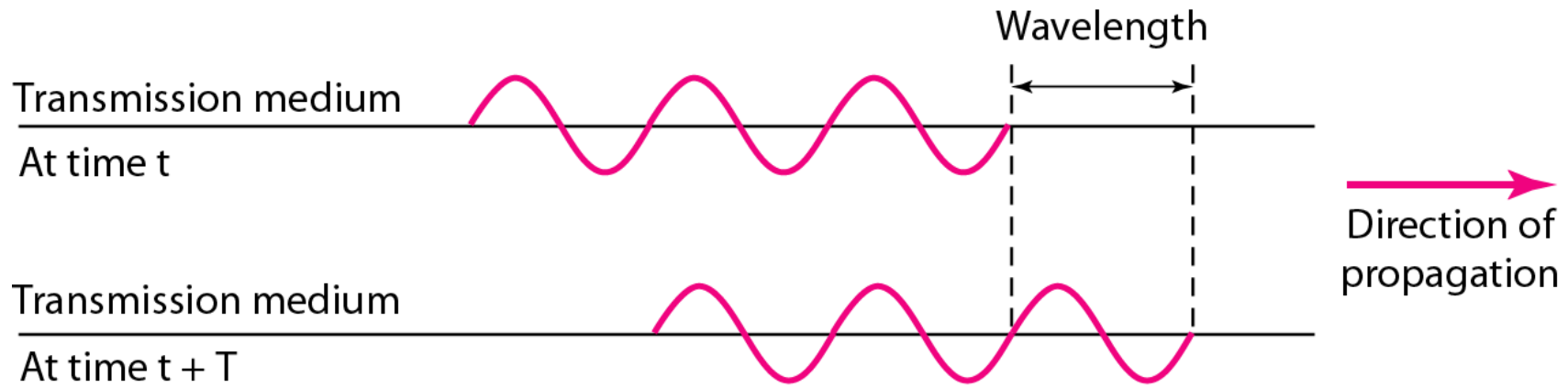
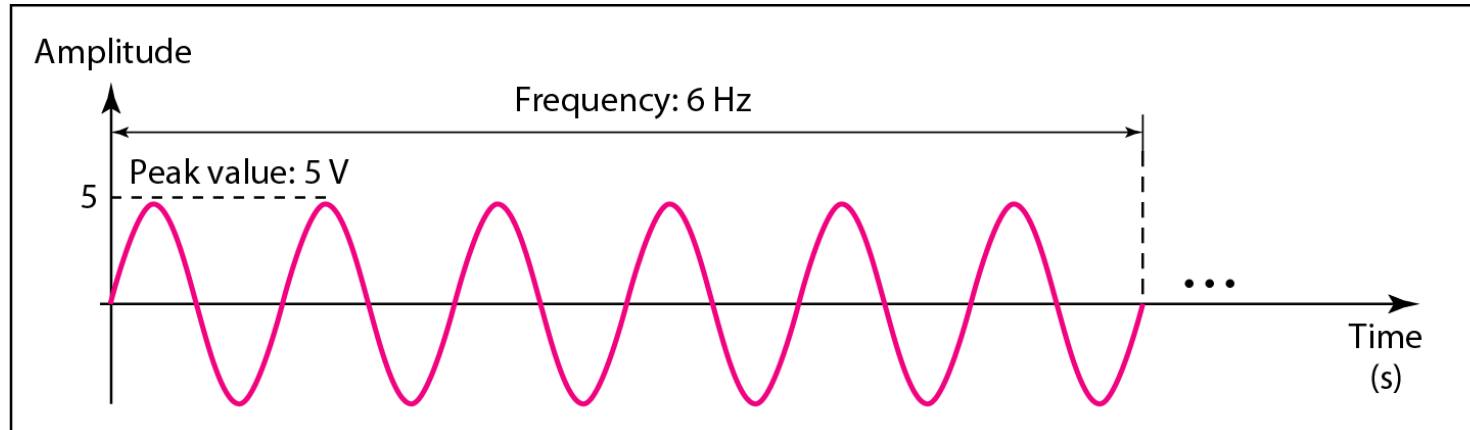
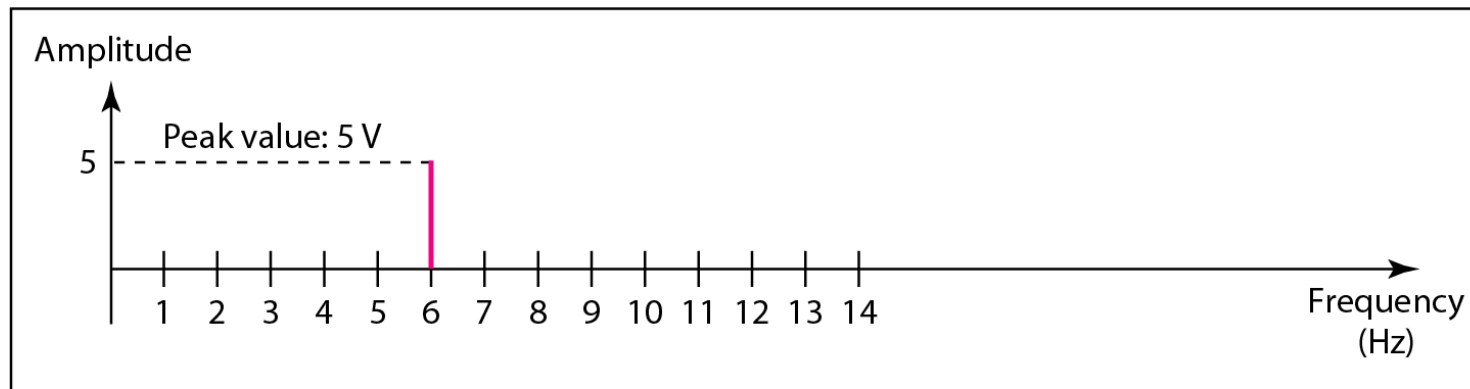


Figure *The time-domain and frequency-domain plots of a sine wave*



a. A sine wave in the time domain (peak value: 5 V, frequency: 6 Hz)



b. The same sine wave in the frequency domain (peak value: 5 V, frequency: 6 Hz)

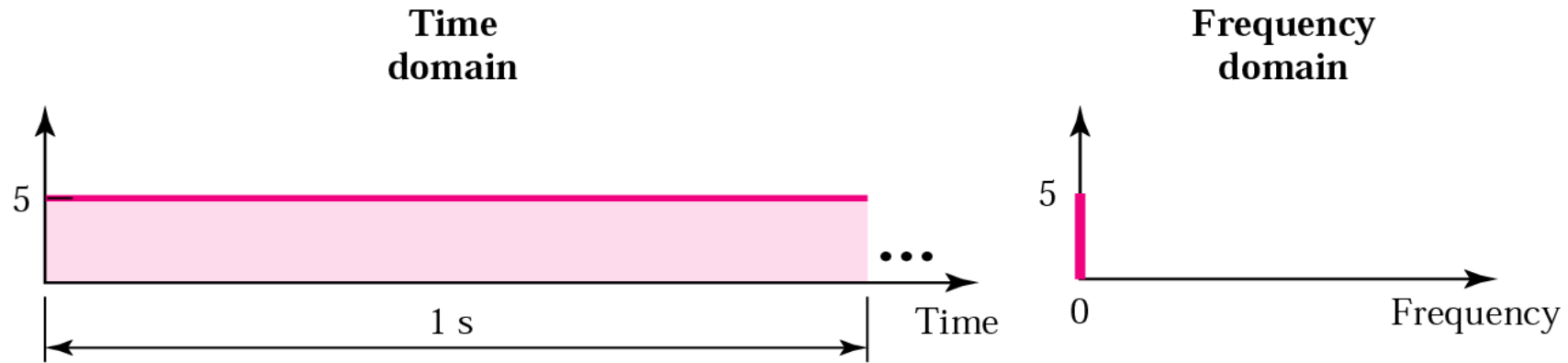
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A complete sine wave in the time domain  
can be represented by one single spike in  
the frequency domain.

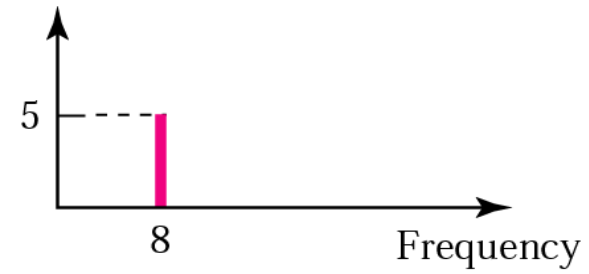
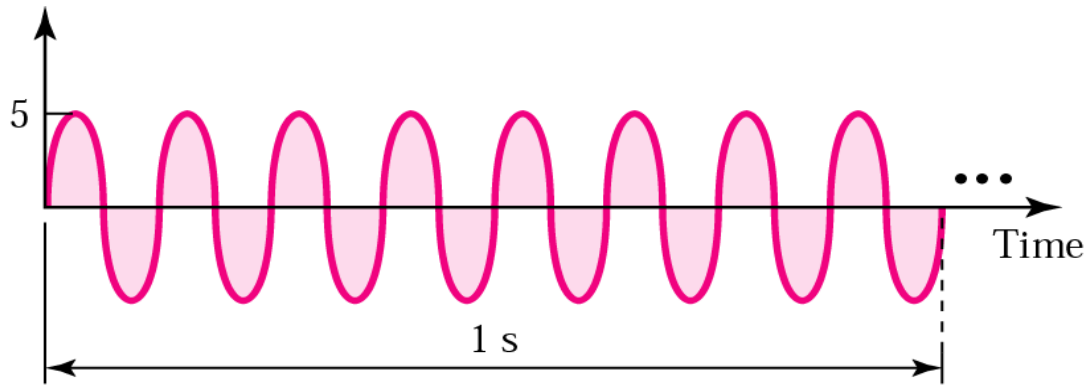
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# Lecture 5

*Time and frequency domains*

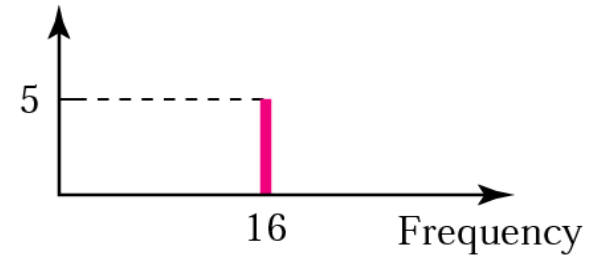
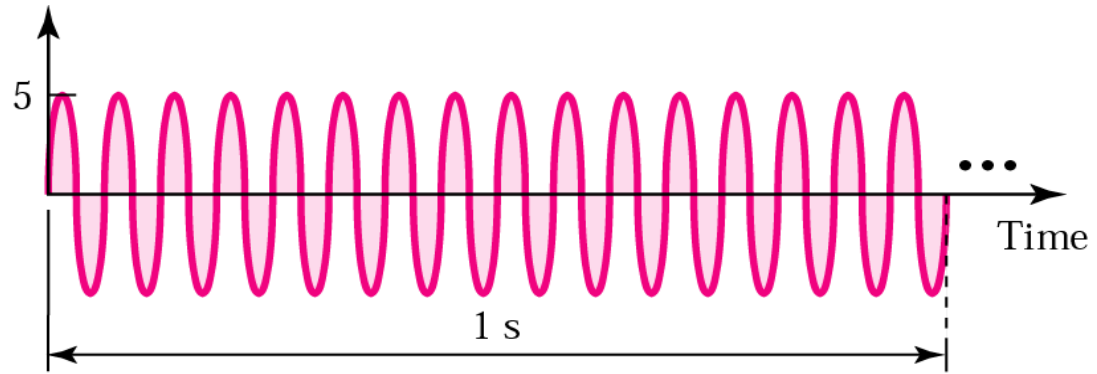


a. A signal with frequency 0



b. A signal with frequency 8





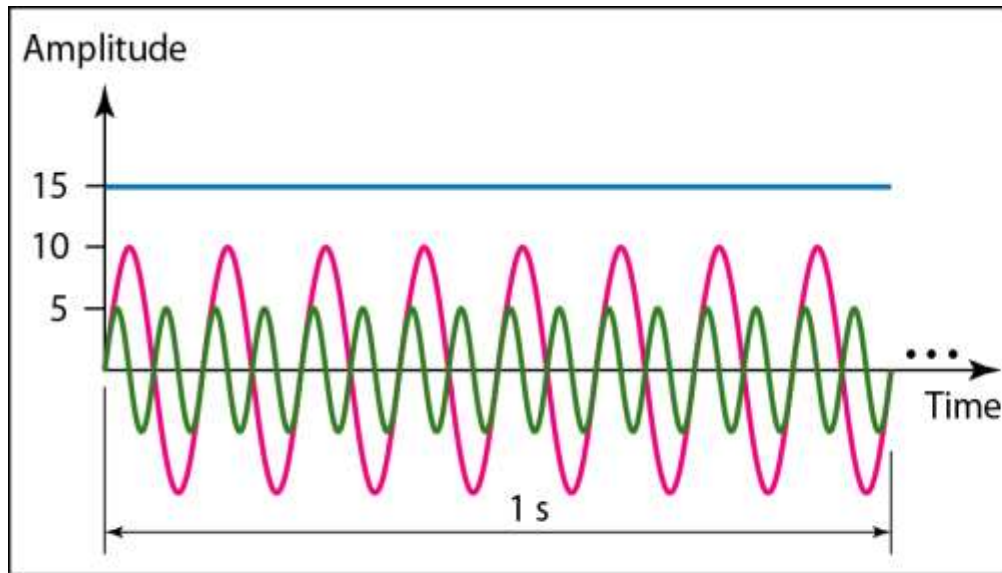
c. A signal with frequency 16



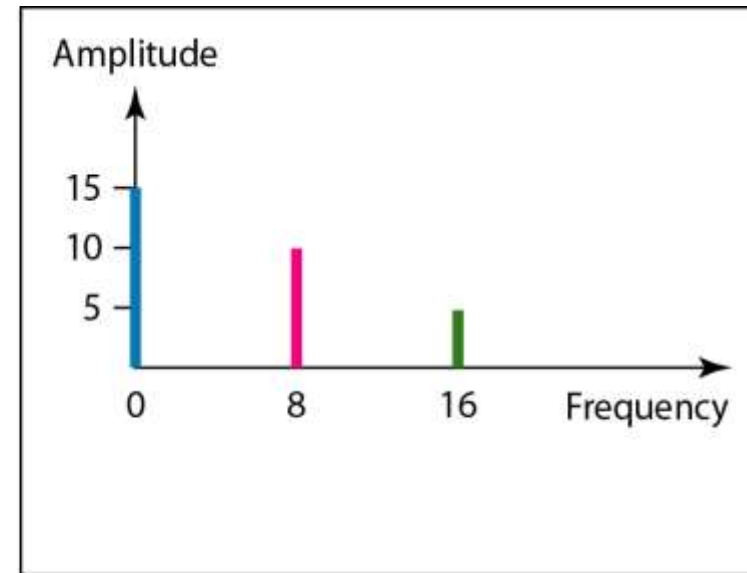
## *Example*

*The frequency domain is more compact and useful when we are dealing with more than one sine wave. For example, Next Figure shows three sine waves, each with different amplitude and frequency. All can be represented by three spikes in the frequency domain.*

Figure *The time domain and frequency domain of three sine waves*



a. Time-domain representation of three sine waves with frequencies 0, 8, and 16

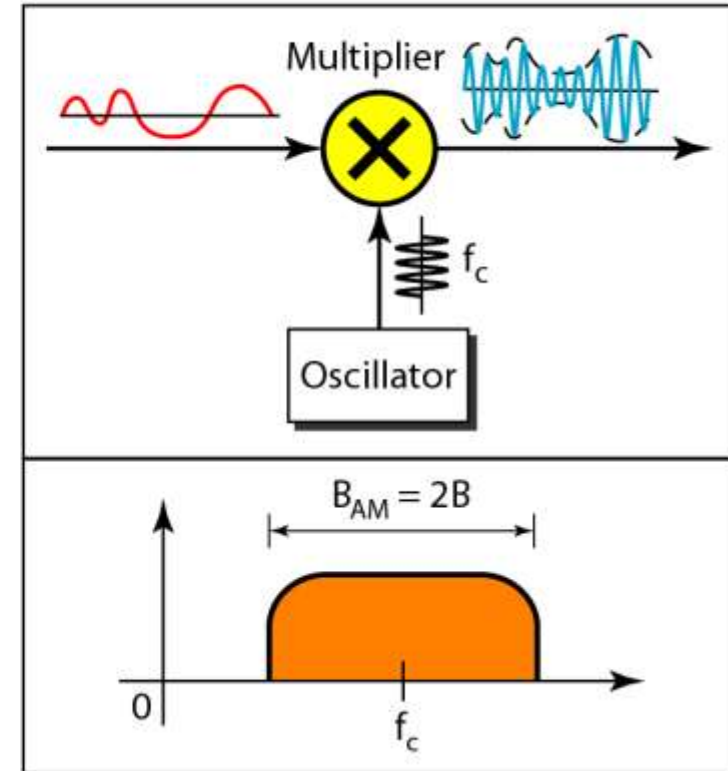
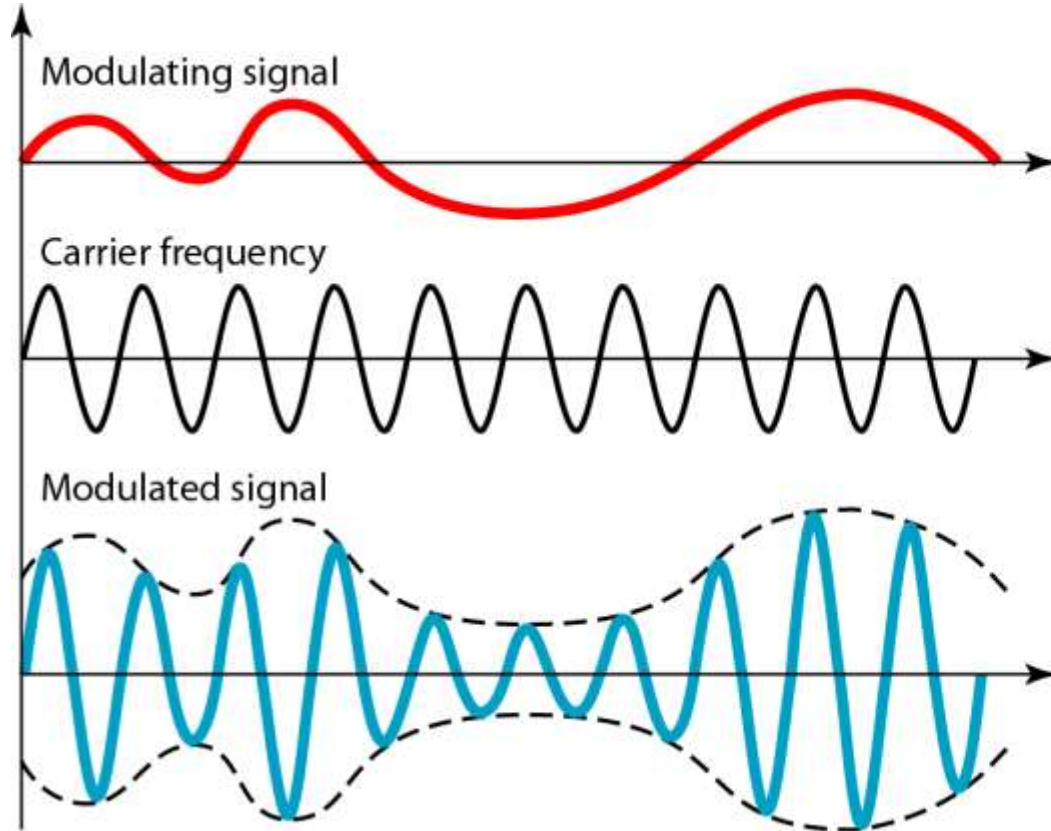


b. Frequency-domain representation of the same three signals

---

A single-frequency sine wave is not useful  
in communication systems;  
we need to send a composite signal, a  
signal made of many simple sine waves.

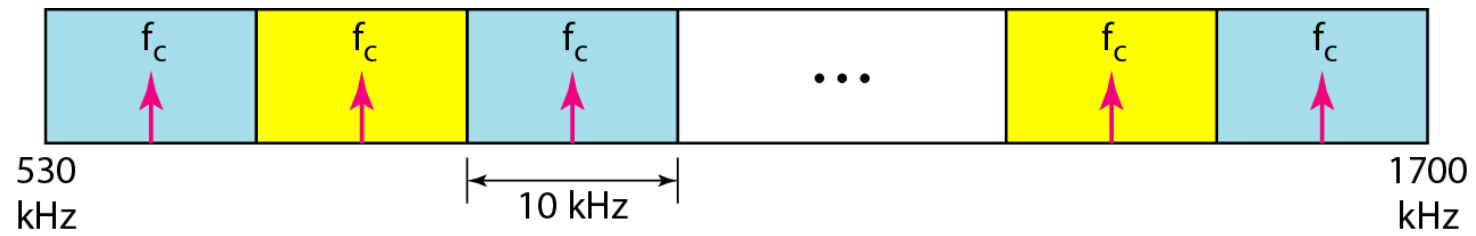
## Example *Amplitude modulation*



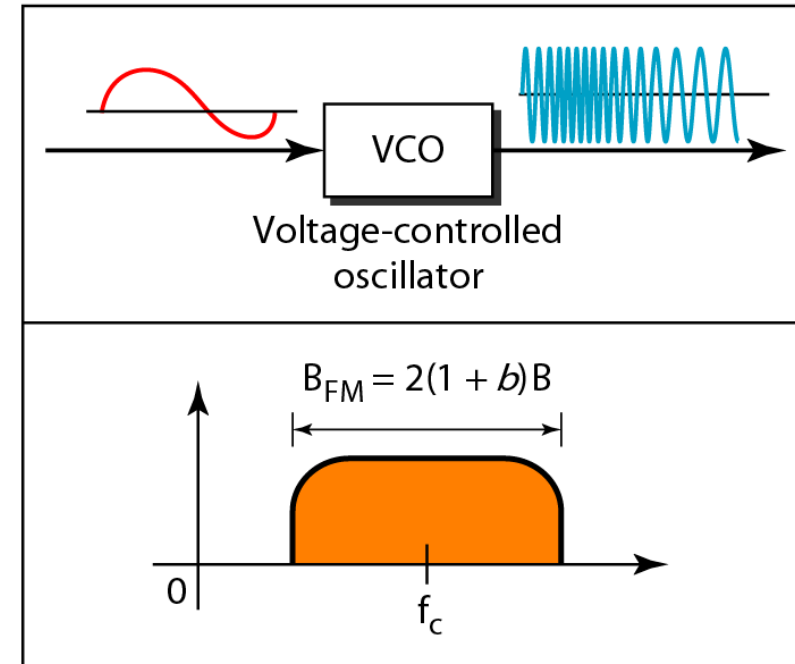
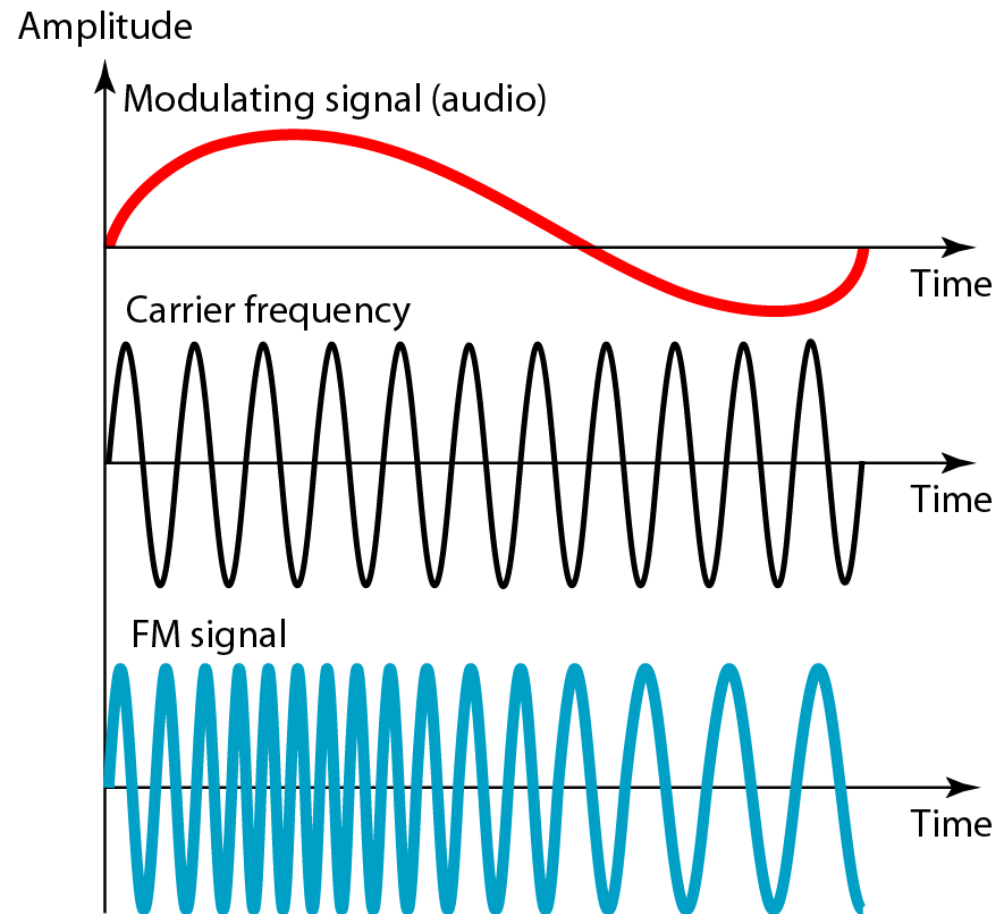
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Figure *AM band allocation*

---



# Figure *Frequency modulation*



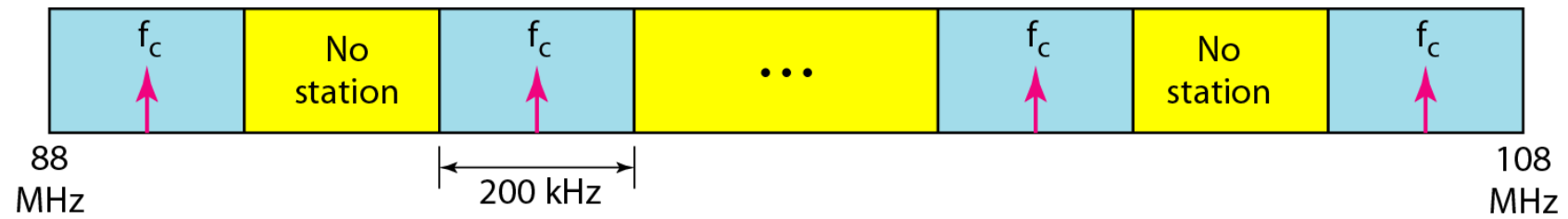
# Lecture 6



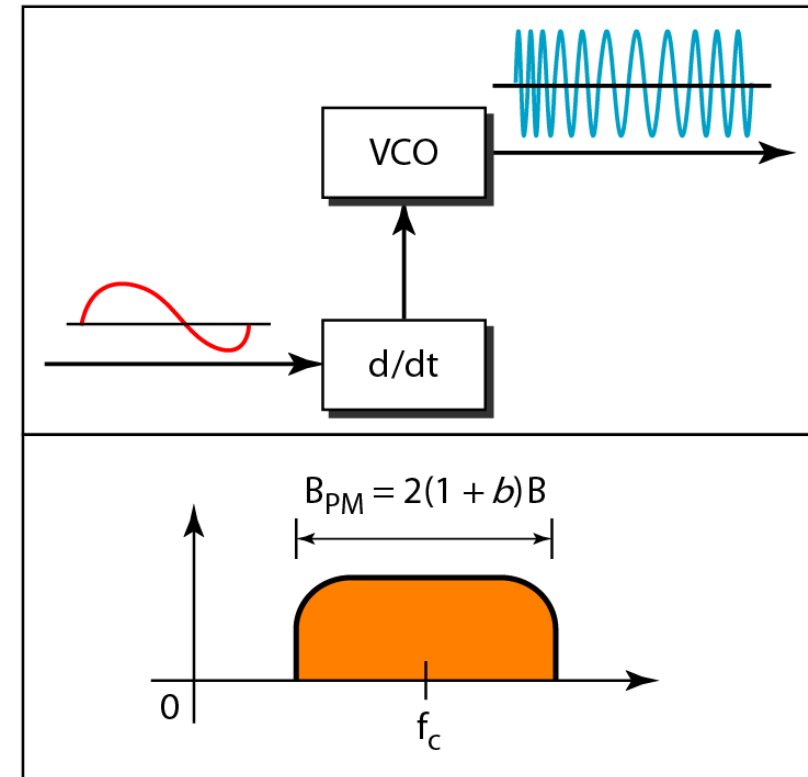
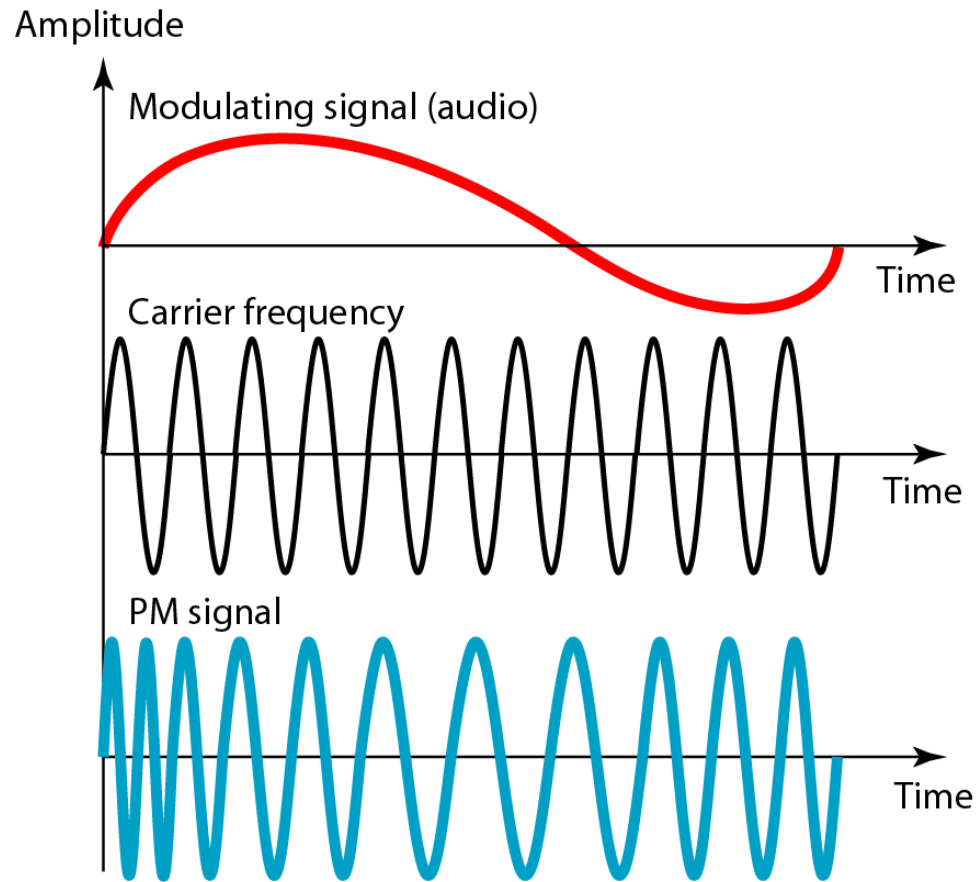
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## Figure *FM band allocation*

---



# Figure *Phase modulation*



---

According to Fourier analysis, any composite signal is a combination of simple sine waves with different frequencies, amplitudes, and phases.

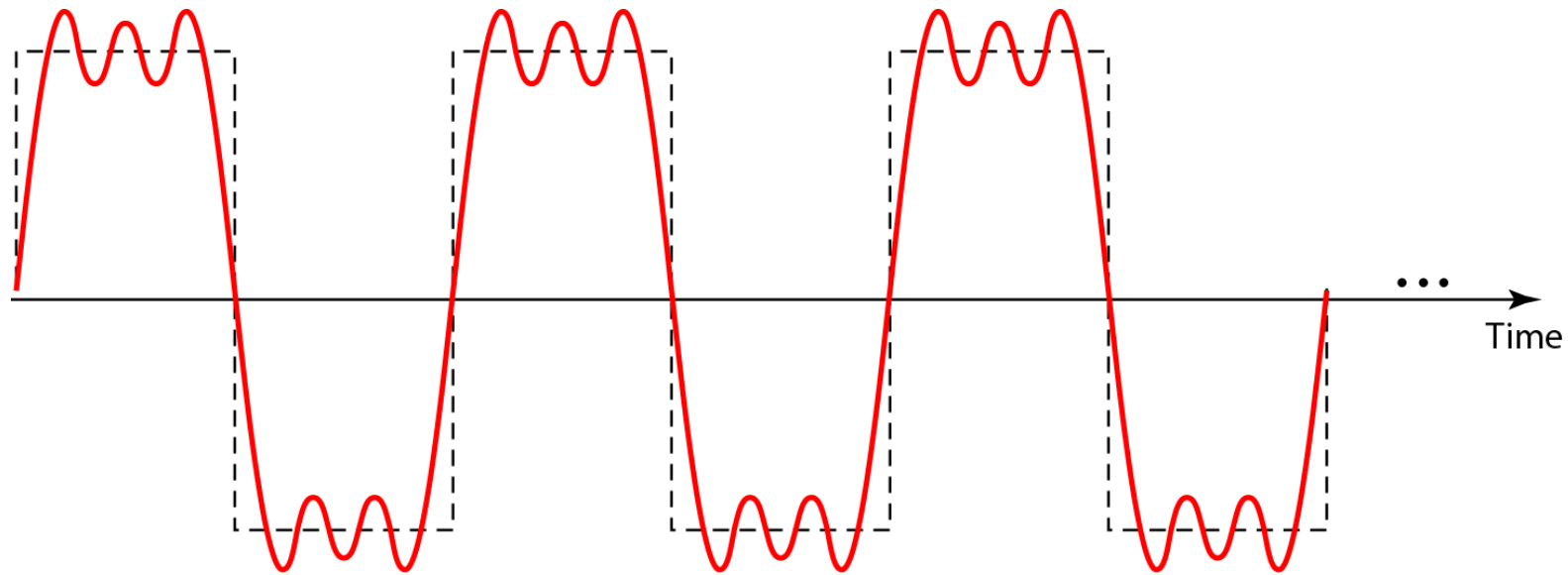
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If the composite signal is periodic, the decomposition gives a series of signals with discrete frequencies;

if the composite signal is nonperiodic, the decomposition gives a combination of sine waves with continuous frequencies.

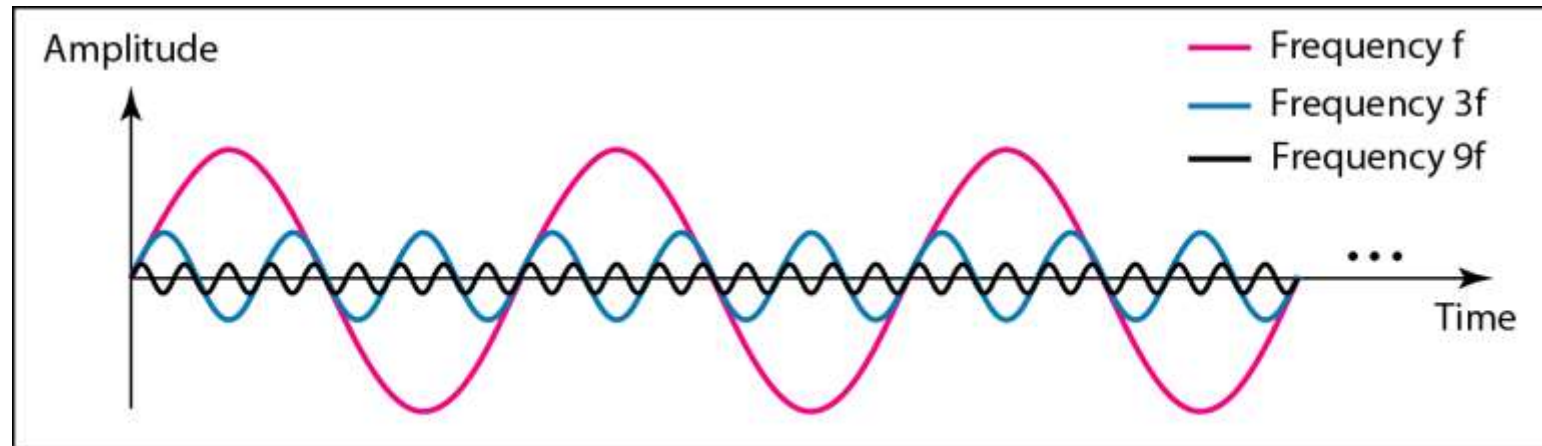
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Figure *A composite periodic signal*

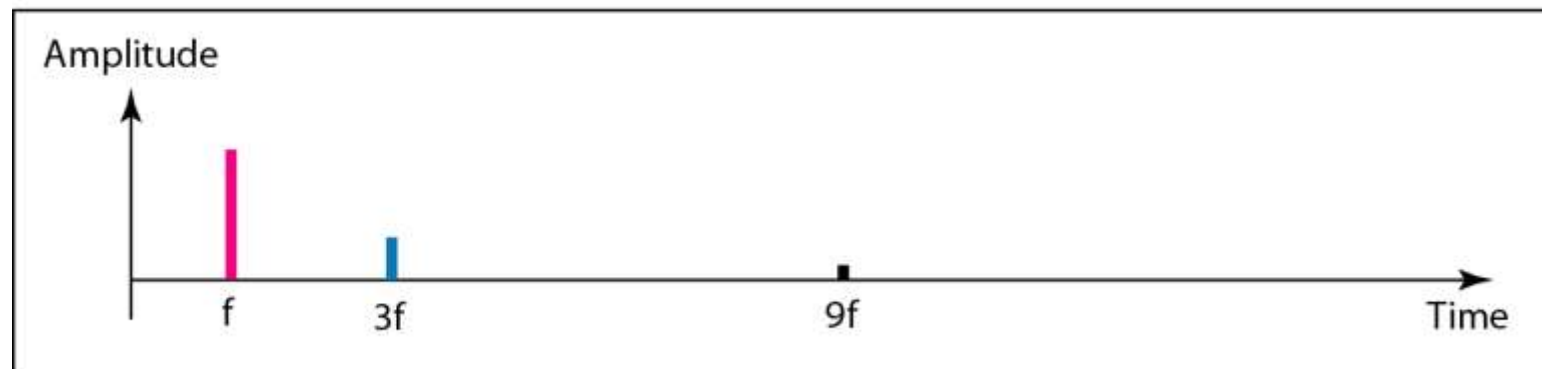


*Above Figure shows a periodic composite signal with frequency  $f$ . This type of signal is not typical of those found in data communications. We can consider it to be three alarm systems, each with a different frequency. The analysis of this signal can give us a good understanding of how to decompose signals.*

Figure *Decomposition of a composite periodic signal in the time and frequency domains*

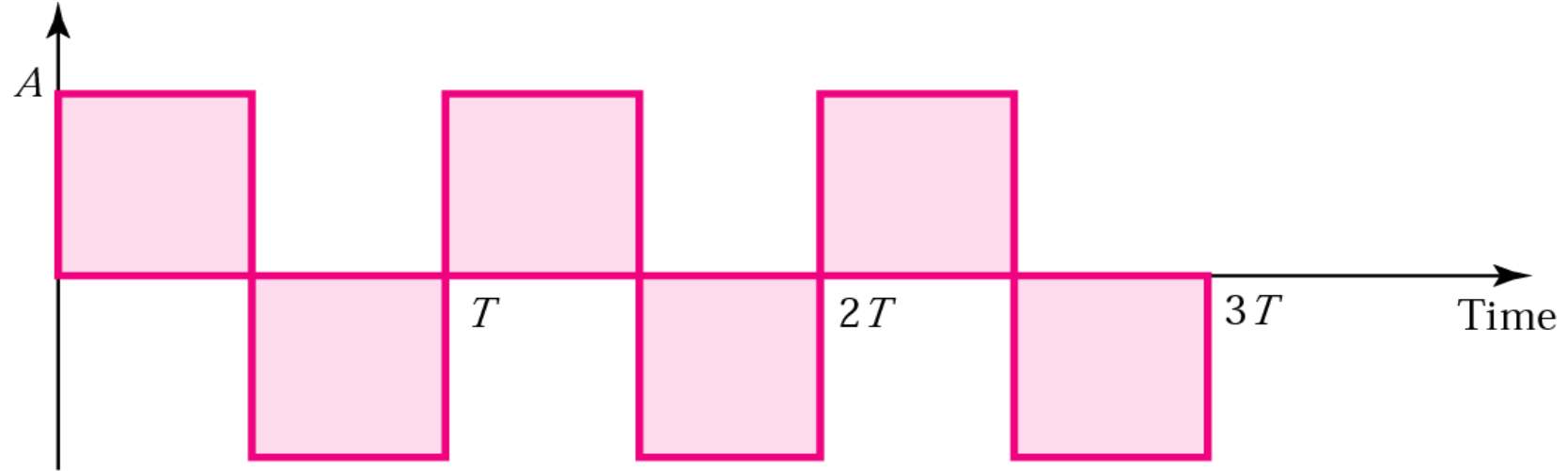


a. Time-domain decomposition of a composite signal

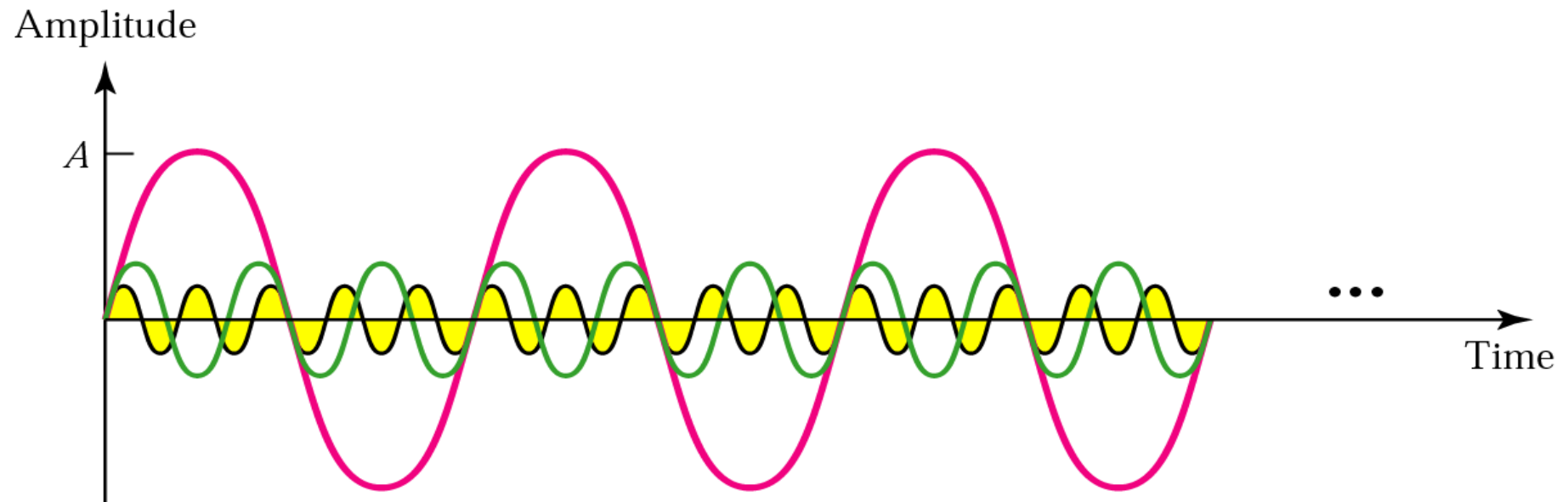


b. Frequency-domain decomposition of the composite signal

*Square wave*

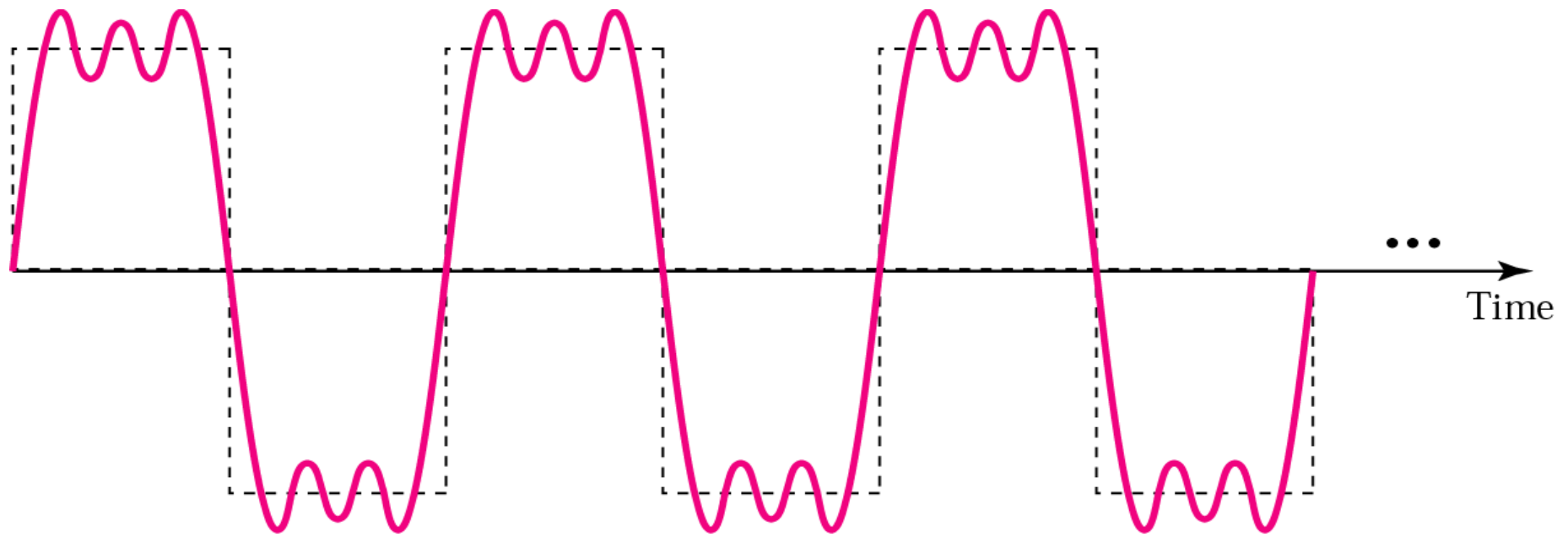


*Three harmonics*



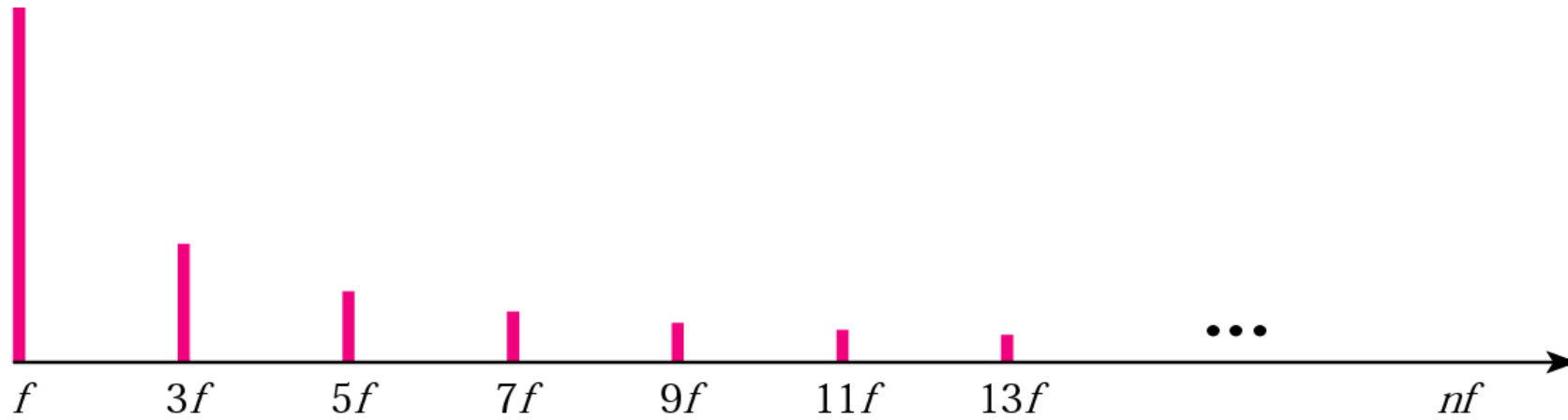


*Adding first three harmonics*



# Lecture SEVEN

*Frequency spectrum comparison*

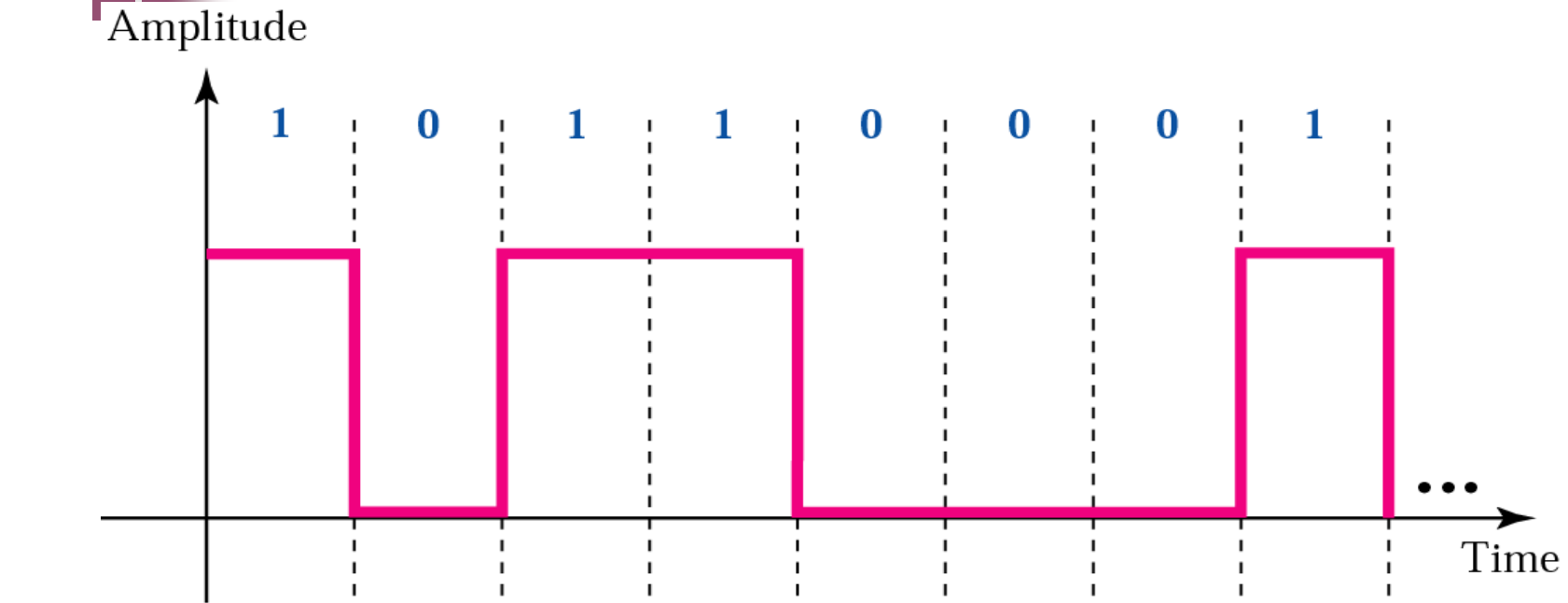


a. Frequency spectrum of a square wave



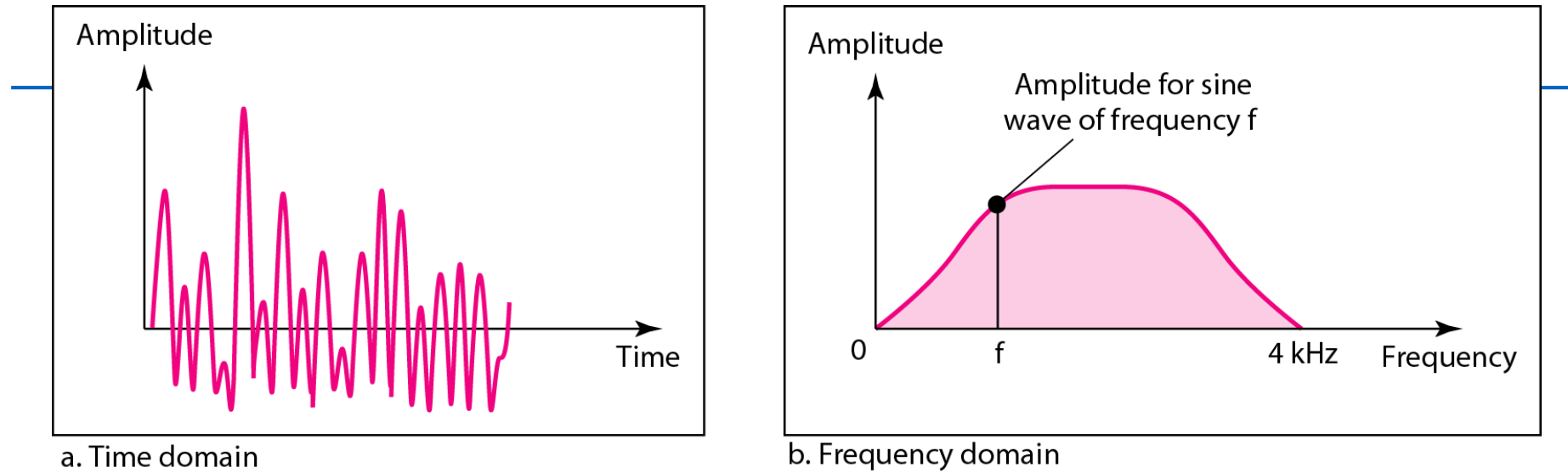
b. Frequency spectrum of an approximation with only three harmonics

## *A digital signal*



*A digital signal is a composite signal with an infinite bandwidth.*

Figure *The time and frequency domains of a nonperiodic signal*

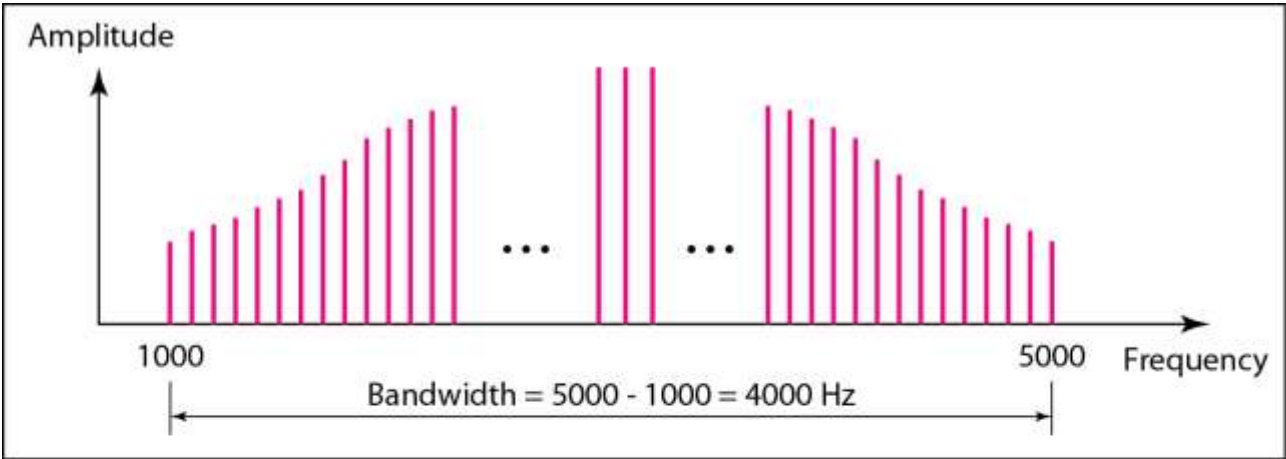


*Above Figure shows a nonperiodic composite signal. It can be the signal created by a microphone or a telephone set when a word or two is pronounced. In this case, the composite signal cannot be periodic, because that implies that we are repeating the same word or words with exactly the same tone.*

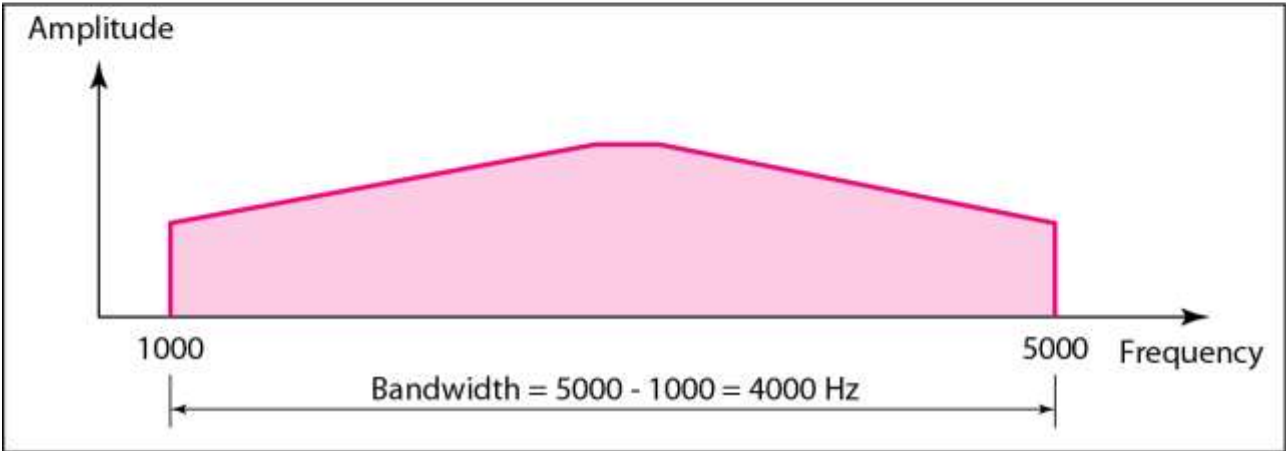
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The bandwidth of a composite signal is the difference between the highest and the lowest frequencies contained in that signal.

Figure *The bandwidth of periodic and nonperiodic composite signals*



a. Bandwidth of a periodic signal



b. Bandwidth of a nonperiodic signal



## Example

*If a periodic signal is decomposed into five sine waves with frequencies of 100, 300, 500, 700, and 900 Hz, what is its bandwidth? Draw the spectrum, assuming all components have a maximum amplitude of 10 V.*

*Solution*

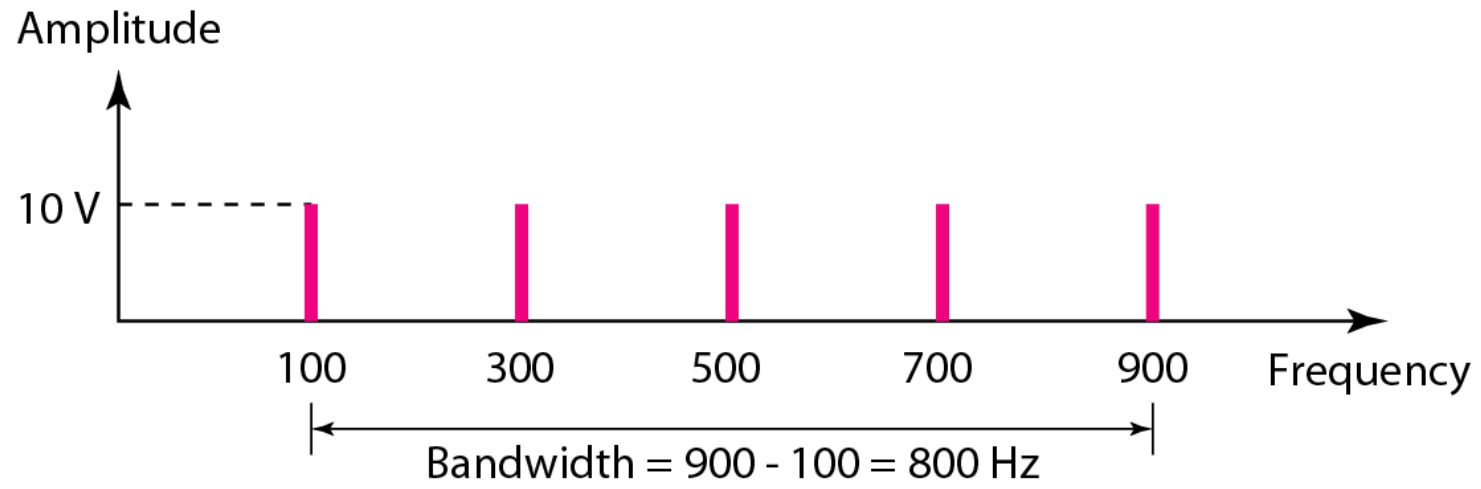
*Let  $f_h$  be the highest frequency,  $f_l$  the lowest frequency, and  $B$  the bandwidth. Then*

$$B = f_h - f_l = 900 - 100 = 800 \text{ Hz}$$

*The spectrum has only five spikes, at 100, 300, 500, 700, and 900 Hz (see next Figure ).*



Figure *The bandwidth for Example*





## *Example*

*A periodic signal has a bandwidth of 20 Hz. The highest frequency is 60 Hz. What is the lowest frequency? Draw the spectrum if the signal contains all frequencies of the same amplitude.*

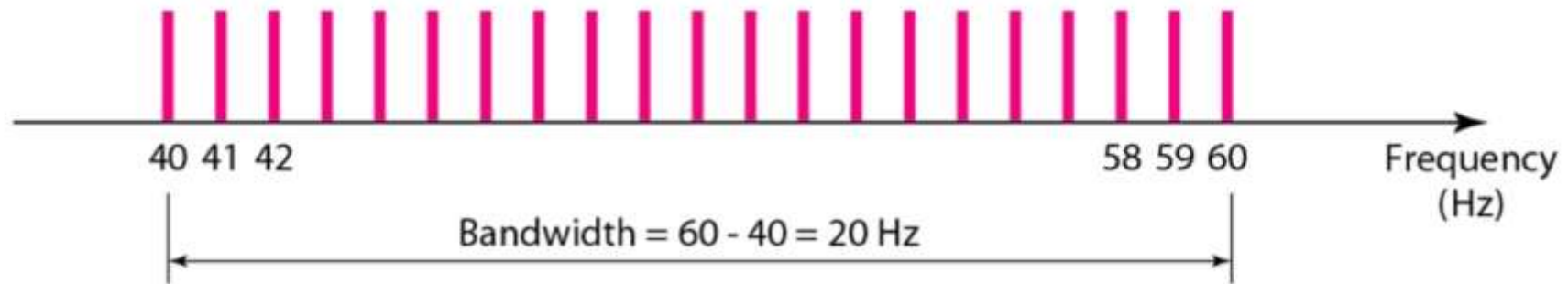
### *Solution*

*Let  $f_h$  be the highest frequency,  $f_l$  the lowest frequency, and  $B$  the bandwidth. Then*

$$B = f_h - f_l \Rightarrow 20 = 60 - f_l \Rightarrow f_l = 60 - 20 = 40 \text{ Hz}$$

*The spectrum contains all integer frequencies. We show this by a series of spikes (see next Figure ).*

**Figure** *The bandwidth for Example*





## *Example*

---

*A nonperiodic composite signal has a bandwidth of 200 kHz, with a middle frequency of 140 kHz and peak amplitude of 20 V. The two extreme frequencies have an amplitude of 0. Draw the frequency domain of the signal.*

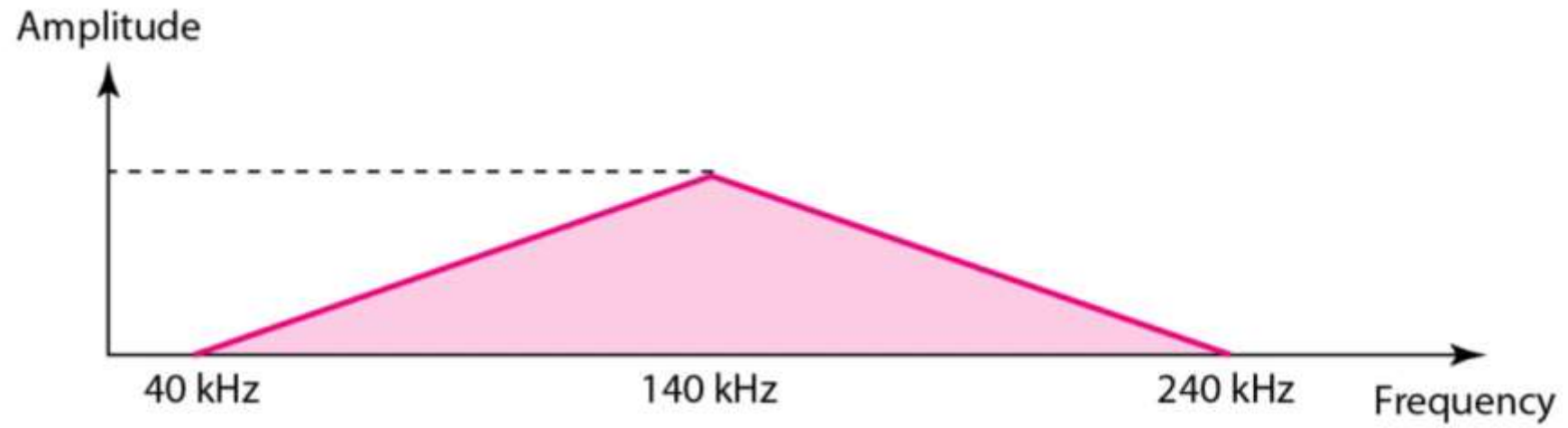
### *Solution*

*The lowest frequency must be at 40 kHz and the highest at 240 kHz. Next Figure shows the frequency domain and the bandwidth.*

---

**Figure** *The bandwidth for Example*

---





## *Example*

---

*An example of a nonperiodic composite signal is the signal propagated by an AM radio station. Each AM radio station is assigned a 10-kHz bandwidth. The total bandwidth dedicated to AM radio ranges from 530 to 1700 kHz.*



## *Example*

---

*Another example of a nonperiodic composite signal is the signal propagated by an FM radio station. Each FM radio station is assigned a 200-kHz bandwidth. The total bandwidth dedicated to FM radio ranges from 88 to 108 MHz.*



# Analog and Digital Communication Systems

There are many kinds of information sources, which can be categorized into two distinct message categories, *analog* and *digital*.

*an **analog communication system** should deliver this waveform with a specified degree of fidelity.*

*a **digital communication system** should deliver data with a specified degree of accuracy in a specified amount of time.*



# Comparisons of Digital and Analog Communication Systems

<b>Digital Communication System</b>	<b>Analog Communication System</b>
<b>Advantage :</b> <ul style="list-style-type: none"><li>● inexpensive digital circuits</li><li>● privacy preserved (data encryption)</li><li>● can merge different data (voice, video and data) and transmit over a common digital transmission system</li><li>● error correction by coding</li></ul>	<b>Disadvantages :</b> <ul style="list-style-type: none"><li>● expensive analog components : L&amp;C</li><li>● no privacy</li><li>● can not merge data from diff. sources</li><li>● no error correction capability</li></ul>
<b>Disadvantages :</b> <ul style="list-style-type: none"><li>● larger bandwidth</li><li>● synchronization problem is relatively difficult</li></ul>	<b>Advantages :</b> <ul style="list-style-type: none"><li>● smaller bandwidth</li><li>● synchronization problem is relatively easier</li></ul>

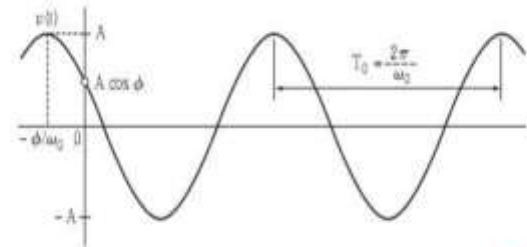
# Brief Chronology of Communication Systems

- 1844 *Telegraph.*
- 1876 *Telephony.*
- 1904 *Radio:*
- 1923-1938 *Television.*
- 1936 Armstrong's case of FM radio
- 1938-1945 *World War II* Radar and microwave systems
- 1948-1950 *Information Theory and coding.* C. E. Shannon
- 1962 *Satellite* communications begins with Telstar I.
- 1962-1966 *High Speed digital communication*
- 1972 Motorola develops *cellular telephone.*

# Signals and Spectra

# Phasors and Line Spectra

$$v(t) = A \cos(\omega_0 t + \varphi)$$



The phasor representation of a sinusoidal signal comes from *Euler's theorem*

$$e^{\pm j\theta} = \cos \theta \pm j \sin \theta$$

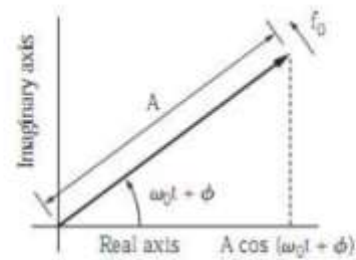
Any sinusoid as the real part of a complex exponential

$$A \cos(\omega_0 t + \varphi) = A \operatorname{Re}[e^{j(\omega_0 t + \varphi)}] = \operatorname{Re}[A e^{j\varphi} e^{j\omega_0 t}]$$

Any sinusoid as the real part of a complex exponential

$$A \cos(\omega_0 t + \varphi) = A \operatorname{Re}[e^{j(\omega_0 t + \varphi)}] = \operatorname{Re}[Ae^{j\varphi} e^{j\omega_0 t}]$$

This is called a **phasor representation**



Only three parameters completely specify a phasor: **amplitude**, **phase angle**, and **rotational frequency**

A suitable frequency-domain description would be the *line spectrum*



*One sided spectra*

Phase angles will be measured with respect to *cosine* waves. Hence, sine waves need to be converted to cosines via the identity

$$\sin \omega t = \cos (\omega t - 90^\circ)$$

We regard *amplitude* as always being a *positive quantity*. When negative signs appear, they must be absorbed in the phase using

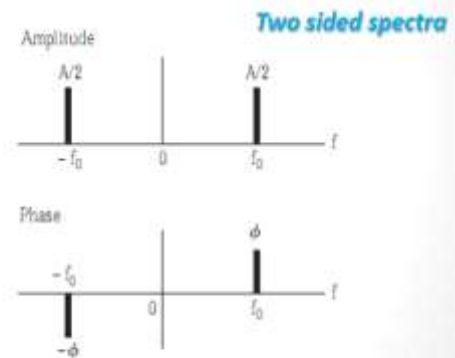
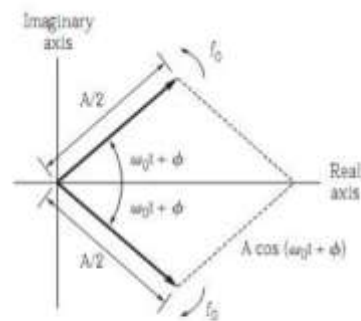
$$-A \cos \omega t = A \cos (\omega t \pm 180^\circ)$$

Recalling that  $\text{Re}[z] = \frac{1}{2}(z + z^*)$

$$\text{If } z = Ae^{j\varphi} e^{j\omega_0 t}$$

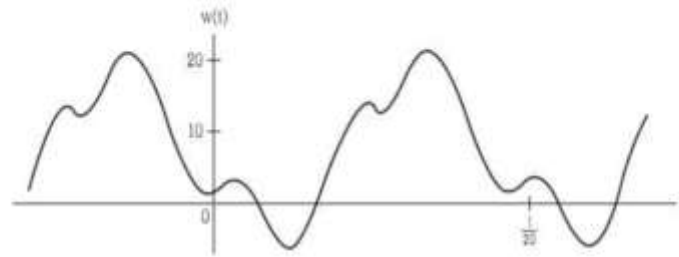
then

$$A \cos(\omega_0 t + \varphi) = \frac{A}{2} e^{j\varphi} e^{j\omega_0 t} + \frac{A}{2} e^{-j\varphi} e^{-j\omega_0 t}$$

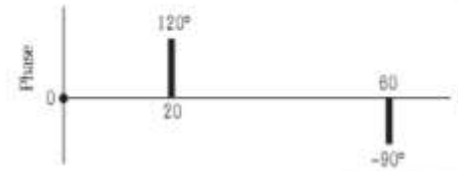
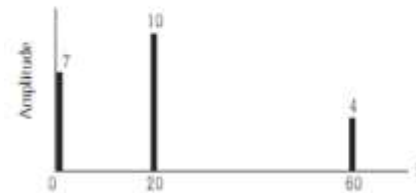


consider the signal

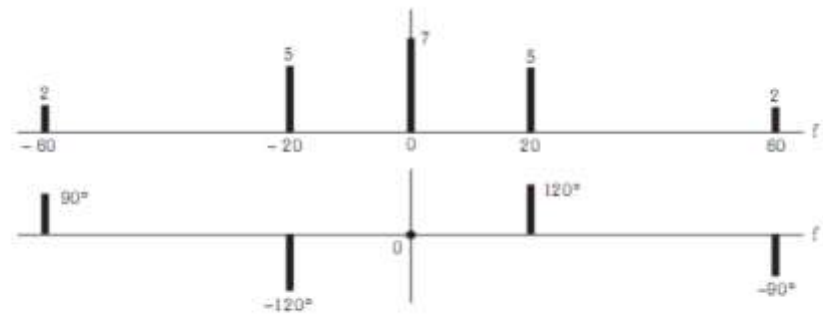
$$w(t) = 7 - 10 \cos(40\pi t - 60^\circ) + 4 \sin 120\pi t$$



$$w(t) = 7 \cos 2\pi 0t + 10 \cos(2\pi 20t + 120^\circ) + 4 \cos(2\pi 60t - 90^\circ)$$







# Lecture ten

# Periodic Signals and Average Power

The **average** value of any function  $v(t)$  is defined as

$$\langle v(t) \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} v(t) dt$$

In case of **periodic signal**

$$\langle v(t) \rangle = \frac{1}{T_o} \int_{t_1}^{t_1+T_o} v(t) dt = \frac{1}{T_o} \int_{T_o} v(t) dt$$

The average power (**normalized**)

$$P = \langle |v(t)|^2 \rangle = \frac{1}{T_o} \int_{T_o} |v(t)|^2 dt$$

The average value of a power signal may be **positive**, **negative**, or **zero**.

# Fourier Series

Let  $v(t)$  be a power signal with period  $T_0 \equiv 1/f_0$ . Its exponential Fourier series expansion is

$$v(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_0 t} \quad n = 0, 1, 2, \dots$$

The series coefficients are related to  $v(t)$  by

$$c_n = \frac{1}{T_0} \int_{T_0} v(t) e^{-j2\pi n f_0 t} dt$$

The coefficients are complex quantities in general, they can be expressed in the **polar form**

$$c_n = |c_n| e^{j \arg c_n}$$

the  **$n^{\text{th}}$**  term of the Fourier series equation being

$$c_n e^{j2\pi n f_0 t} = |c_n| e^{j \arg c_n} e^{j2\pi n f_0 t}$$

$|c(nf_0)|$  represents the *amplitude spectrum* as a function of  $f$ , and  $\arg c(nf_0)$  represents the *phase spectrum*.

Three important spectral properties of periodic power signals are listed below.

1. All frequencies are integer multiples or harmonics of the fundamental frequency  $f_0 = 1/T_0$ . Thus the spectral lines have *uniform spacing*  $f_0$ .

2. The DC component equals the *average value* of the signal, since setting  $n = 0$

$$c(0) = \frac{1}{T_0} \int_{T_0} v(t) dt = \langle v(t) \rangle$$

3. If  $v(t)$  is a real (noncomplex) function of time, then

$$c_{-n} = c_n^* = |c_n| e^{j \arg c_n}$$

With replacing  $n$  by  $-n$

$$|c(-nf_0)| = |c(nf_0)| \quad \arg c(-nf_0) = -\arg c(nf_0)$$

which means that the *amplitude spectrum* has *even symmetry* and the *phase spectrum* has *odd symmetry*.

## trigonometric Fourier Series *a one-sided spectrum*

$$v(t) = c_0 + \sum_{n=1}^{\infty} |2c_n| \cos(2\pi n f_0 t + \arg c_n)$$

or

$$v(t) = c_0 + \sum_{n=1}^{\infty} a_n \cos 2\pi n f_0 t + b_n \sin 2\pi n f_0 t$$

$$a_n = \operatorname{Re}[c_n] \text{ and } b_n = \operatorname{Im}[c_n]$$

These sinusoidal terms represent a set of *orthogonal basis functions*,

Functions  $v_n(t)$  and  $v_m(t)$  are orthogonal over an interval from  $t_1$  to  $t_2$  if

$$\int_{t_1}^{t_2} v_n(t) v_m(t) dt = \begin{cases} 0 & n \neq m \\ K & n = m \end{cases} \quad \text{with } K \text{ a constant}$$



The integration for  $c_n$  often involves a phasor average in the form

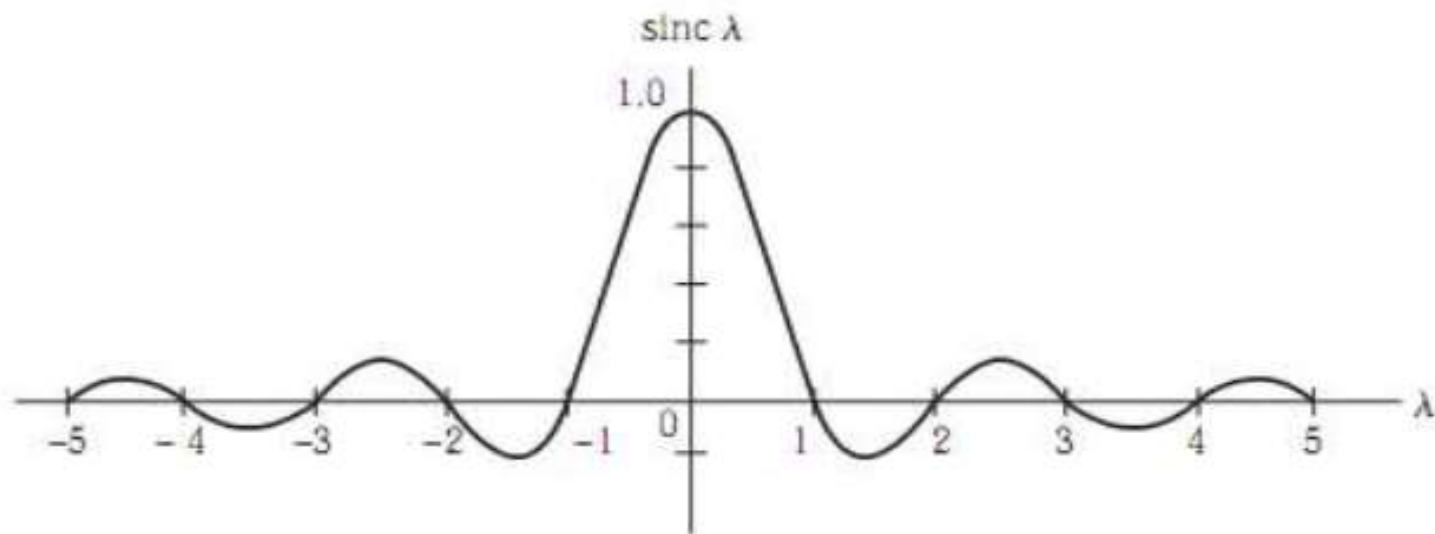
$$\frac{1}{T_0} \int_{-T/2}^{T/2} e^{j2\pi f t} dt = \frac{1}{j2\pi f T} (e^{j\pi f T} - e^{-j\pi f T}) = \frac{1}{\pi f T} \sin \pi f T$$

we'll now introduce the *sinc* function defined by

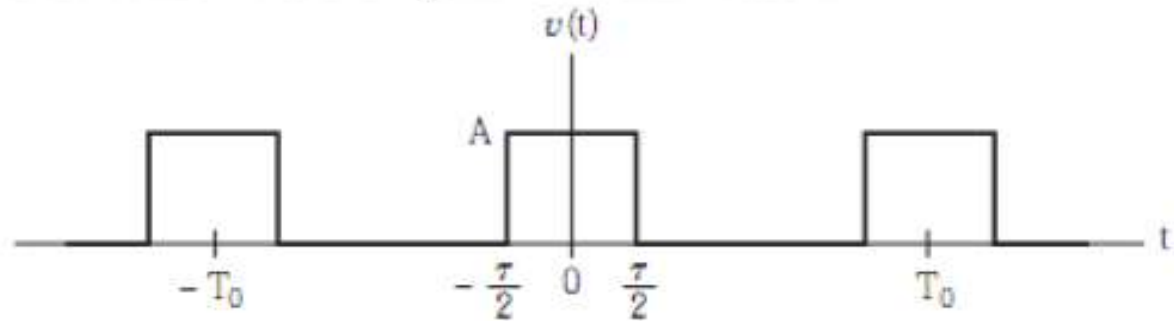
$$\text{sinc } \lambda \triangleq \frac{\sin \pi \lambda}{\pi \lambda}$$

$\text{sinc } \lambda$  is an even function of  $\lambda$  having its peak at  $\lambda = 0$  and zero crossings at all other integer values of  $\lambda$ , so

$$\text{sinc } \lambda = \begin{cases} 1 & \lambda = 0 \\ 0 & \lambda = \pm 1, \pm 2, \dots \end{cases}$$



**EXAMPLE:** Rectangular Pulse Train



$$v(t) = \begin{cases} A & |t| < \tau/2 \\ 0 & |t| > \tau/2 \end{cases}$$

To calculate the Fourier coefficients

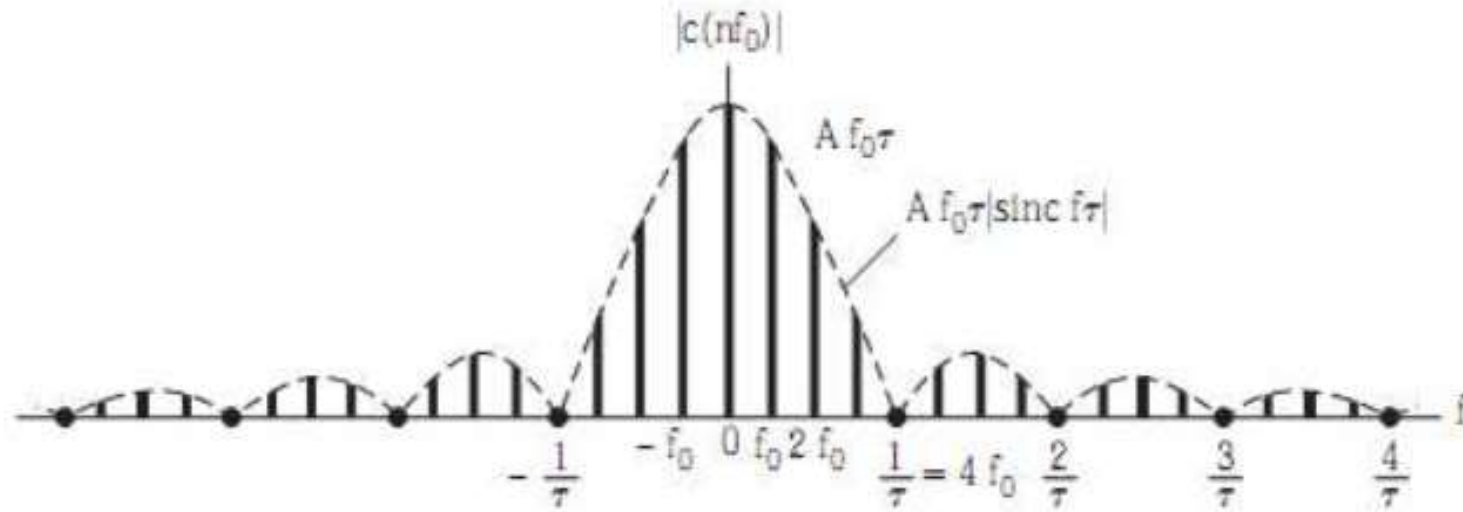
$$\begin{aligned} c_n &= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} v(t) e^{-j2\pi n f_0 t} dt = \frac{1}{T_0} \int_{-\tau/2}^{\tau/2} A e^{-j2\pi n f_0 t} dt \\ &= \frac{A}{-j\pi n f_0 T_0} (e^{-j\pi n f_0 \tau} - e^{+j\pi n f_0 \tau}) = \frac{A}{T_0} \frac{\sin \pi n f_0 \tau}{\pi n f_0} \end{aligned}$$

Multiplying and dividing by  $\tau$  finally gives

$$c_n = \frac{A\tau}{T_0} \text{sinc } n f_0 \tau$$



The **amplitude spectrum** obtained from  $|c(nf_0)| = |c_n| = Af_0 \tau |\text{sinc } nf_0 \tau|$



for the case of  $\tau/T_0 = \tau f_0 = 1/4$

We construct this plot by drawing the continuous function  $Af_0 \tau |\text{sinc } n\tau|$  as a **dashed curve**, which becomes the **envelope of the lines**.

The spectral lines at  $\pm 4f_0$ ,  $\pm 8f_0$ , and so on, are “missing” since they fall precisely at **multiples of  $1/\tau$**  where the envelope **equals zero**.

The **dc component** has amplitude  $c(0) = A\tau/T_0$  which should be recognized as the average value of  $v(t)$ .

Incidentally,  $\tau/T_0$  equals the ratio of “on” time to period, frequently designated as the **duty cycle** in pulse electronics work

# Transmission Lines

# Transmission lines

Used to transmit signals point-to-point •

## Requirements•

- Preserve signal fidelity (low distortion)
  - signal voltage levels
  - signal bandwidth
  - signal phase / timing properties
- Minimum of radiation (EMI)
- Minimum of crosstalk

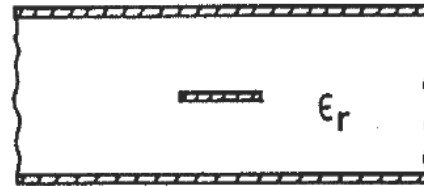
## Parameters of interest•

- Useful frequencies of operation ( $f_1 - f_2$ )
- Attenuation (dB @ MHz)
- Velocity of propagation or delay ( $v_p = X$  cm/ns,  $D = Y$  ns/cm)
- Dispersion ( $v_p(f)$ , frequency-dependent propagation velocity)
- Characteristic impedance ( $Z_o, \Omega$ )
- Size, volume, weight
- Manufacturability or cost (tolerances, complex geometries)

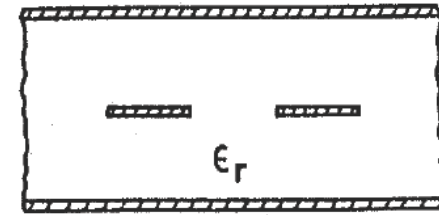
# Transmission lines



RECTANGULAR WAVEGUIDE

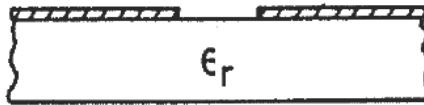


STRIPLINE

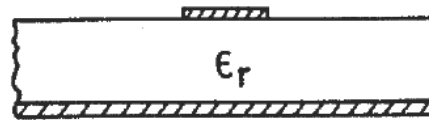


COUPLED STRIPLINES

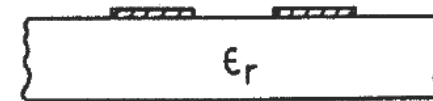
types •  
es•



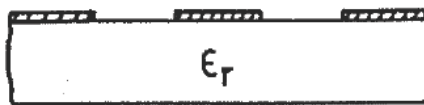
SLOTLINE



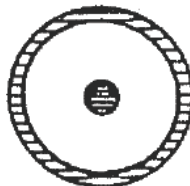
MICROSTRIP LINE



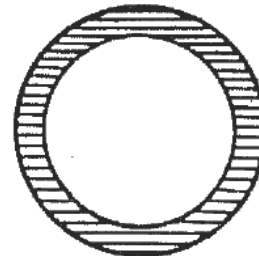
COPLANAR STRIPS



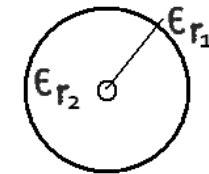
COPLANAR WAVEGUIDE



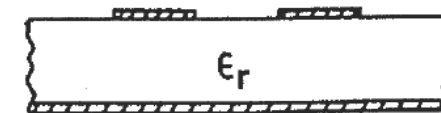
COAXIAL  
LINE



CIRCULAR  
WAVEGUIDE



DIELECTRIC  
WAVEGUIDE



COUPLED MICROSTRIP  
LINES

# Transmission lines

## Transmission line types •

All depend on electromagnetic phenomena •  
Electric fields, magnetic fields, currents

## EM analysis tells us about •

attenuation vs. frequency  $\alpha$   
propagation velocity vs. frequency  $\beta$   
characteristic impedance  $Z_0$   
relative dimensions

## Parameters contributing to these characteristics •

conductivity of metals  $\sigma$   
real part of relative permittivity  $\epsilon_r'$   
imaginary part of relative permittivity  $\epsilon_r''$   
relative permeability (usually  $\sim 1$ )  $\mu_r$

## structure dimensions

height  $h$   
width  $w$   
thickness  $t$

# Transmission lines

Effects of transmission line parameters on system performance•

Attenuation vs. frequency•

Attenuation – reduces signal amplitude, reduces noise margin

Freq-dependent attenuation – reduces higher frequency components, increasing  $T_r$

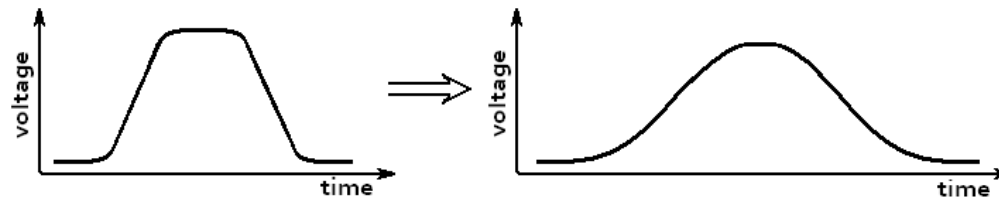
Propagation velocity vs. frequency•

velocity – determines propagation delay between components

reduces max operating frequency

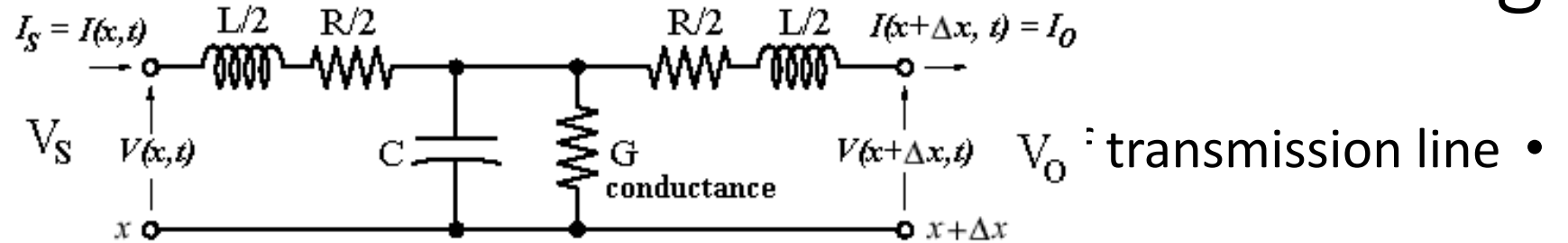
Freq-dependent velocity – distorts signal shape (dispersion)

may broaden the pulse duration



Impedance – ratio of  $V/I$  or  $E/H$   
determines drive requirements  
relates to electromagnetic interference (EMI)  
mismatches lead to signal reflections

# Transmission line modeling



$V_O/V_S = e^{-\gamma z}$  propagation along z axis  
 It can be shown that for a sinusoidal signal,  $\omega = 2\pi f$ ,  $V_s = A e^{j(\omega t + \phi)}$ •

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} \text{ in plane - wave propagation}$$

is analogous to •

- where  $\gamma$  is complex,  $\gamma = \alpha + j\beta$  is the *propagation constant*
- the real part,  $\alpha$  is the *attenuation constant* [Np / m : Np = Nepers]•
- the imaginary part,  $\beta$ , is the *phase constant* [rad / m : rad = radians]•

$$V_O/V_S = e^{-\alpha z} e^{-j\beta z}$$

amplitude change

phase change

# Transmission line modeling

$$\gamma = j\beta = j\omega\sqrt{LC}$$

For a loss-less transmission line ( $R \rightarrow 0, G \rightarrow 0$ ) •

$$V_o = A e^{j(\omega t + \phi)} e^{-\gamma z} = A e^{j(\omega t + \phi - \omega\sqrt{LC}z)}$$

$$t = z\sqrt{LC} \quad \text{or} \quad z/t = 1/\sqrt{LC} \quad \text{so}$$

$$v_p = 1/\sqrt{LC} \quad \text{or} \quad v_p = 1/\sqrt{\mu\epsilon}$$

for the argument to be constant requires

therefore the propagation velocity is

Similarly, for the general case ( $R, G > 0$ ) •

$$\alpha = Re \left\{ \sqrt{(R + j\omega L)(G + j\omega C)} \right\}$$

The characteristic impedance,  $Z_o$  is •

$$Z_o = V_s/I_s = \sqrt{(R + j\omega L)/(G + j\omega C)}$$

and for the low-loss case ( $R \rightarrow 0, G \rightarrow 0$ )

$$Z_o = \sqrt{L/C} \quad \text{or} \quad Z_o = \sqrt{\mu/\epsilon}$$





# Transmission line modeling

Ohmic losses in a printed circuit trace  
 Consider a trace 10 mils wide (0.010") or  $W = 254 \mu\text{m}$   
 2" long (2.000") or  $L = 50.8 \text{ mm}$ , made with 1 ounce copper,  
 $T = 1.35 \text{ mil (0.00135")}$  or  $T = 34.3 \mu\text{m}$   
 (1 oz. = weight per square foot)

For this 2" trace the DC resistance is  $R = 97.4 \text{ m}\Omega$  •

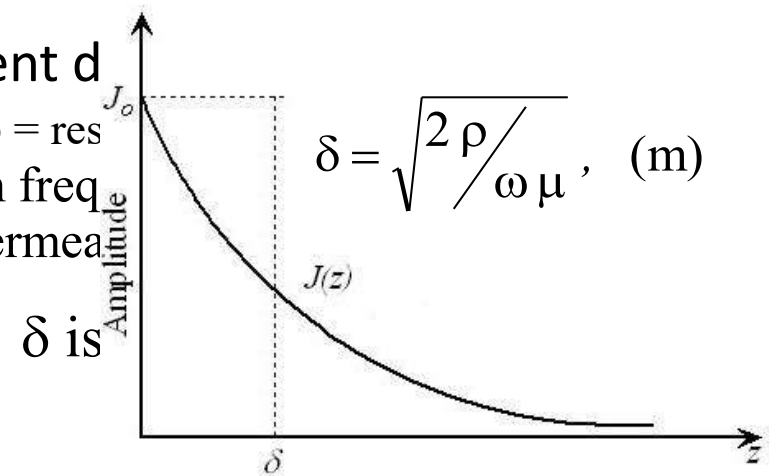
Skin effect •

At DC, the current is uniformly distributed through the conductor •

At higher frequencies, the current density,  $J$ , is highest on the surface and decays exponentially with distance from the surface (due to inductance) •

The average current  $d$

where  $\rho = \text{res}$   
 $\omega = \text{radian freq}$   
 $\mu = \text{magnetic permea}$



$\delta$  is

# Ohmic losses in a printed circuit trace

## transmission line modeling

Consider a copper trace  $W = 10$  mils,  $T = 1.35$  mils,  $L = 2$  mils

Find  $f_s$  such that  $\delta = T/2 = 17.1 \mu\text{m}$

For copper  $\rho = 1.67 \times 10^{-6} \Omega\text{-cm}$

$f_s = \rho / \delta^2$   $\left\{ \begin{array}{l} \text{---} \\ \text{---} \\ \text{---} \end{array} \right.$   $\left. \begin{array}{l} \uparrow \\ \text{---} \\ \downarrow \end{array} \right\}$   $\left. \begin{array}{l} \delta \\ T \\ R(\text{DC}) \end{array} \right\}$   $5 \text{ MHz}$

Therefore  $\delta$   $\left\{ \begin{array}{l} \text{---} \\ \text{---} \\ \text{---} \end{array} \right.$   $\left. \begin{array}{l} \uparrow \\ \text{---} \\ \downarrow \end{array} \right\}$   $\left. \begin{array}{l} \delta \\ T \\ R(\text{DC}) \end{array} \right\}$

for  $f > f_s$ , R increases as  $\sqrt{f}$

Proximity effect •

AC currents follow the path of least impedance •

The path of least inductance causes currents to flow near its return path

This effect applies only to AC ( $f > 0$ ) signals and the effect saturates at relatively low frequency.

Result is a further increase in R

current travels on inner surface

Signal trace



# Ohmic losses in a printed circuit trace • Transmission line modeling

For the 2" long trace, ( $W = 10$  mils.  $T = 1.35$  mils).

find  $f_p$  such

$$f_p = \rho \delta$$



for  $I < I_p$ ,  $R \approx R(DC)$  •

for  $f > f_s$ ,  $R$  increases as  $\sqrt{f}$  •

Since AC currents flow on the metal's surface (skin effect) and •  
on the surface near its return path (proximity effect), plating  
traces on a circuit board with a very good conductor (e.g.,  
silver) will not reduce its ohmic loss.

Conductor loss e;  
 Consider the  
 transmiss



eling

Find the signal atte  
 1500 MHz if the 1

**BROADSIDE-COUPLED  
 TRANSMISSION LINE \***

are 17 mils wide, the substrate's relative dielectric constant  
 is 3.5, and the characteristic impedance ( $Z_0$ ) is  $62 \Omega$ .

$$\alpha(\text{Np/m}) = \text{Re} \left\{ \sqrt{(R + j\omega L) j\omega C} \right\} = \text{Re} \left\{ \sqrt{-\omega^2 LC + j\omega RC} \right\}$$

or  $\alpha(\text{Np/m}) = \text{Re} \left\{ \sqrt{R_x + jI_x} \right\}$  where  $R_x = -\omega^2 LC$ ,  $I_x = \omega RC$

or  $\alpha(\text{Np/m}) = \sqrt{M_x} \cos(P_x/2)$  and  $M_x = \sqrt{R_x^2 + I_x^2}$  and  $P_x = \tan^{-1}(I_x/R_x)$

$$\alpha(\text{dB/m}) = 8.686 \alpha(\text{Np/m})$$

So to solve for  $\alpha(\text{dB/m})$  first find per unit length values for  $R, L, C$

\* Also known as the "parallel-plane transmission line" or the "double-sided parallel strip line"



$$R = \frac{\rho L}{\delta W}$$


where  $\rho$  (copper) =  $1.67 \times 10^{-8} \Omega\text{-m}$ ,  $L = 1 \text{ m}$ ,  $W = 432 \mu\text{m}$ ,  
and  
 $\delta$  is the 1500-MHz skin depth.

For a 1-m length of transmission line ( $L = 1 \text{ m}$ ),  $W = 17$   
mils  
( $W = 432 \mu\text{m}$ ) and  $\delta = 1.71 \mu\text{m}$ , we get  $R = 22.6 \Omega/\text{m}$ .

To find  $C$  and  $L$  over a 1-m length, we know  
 $v_p = c/\sqrt{\epsilon_r} = 1.6 \times 10^8 \text{ m/s}$  and  $v_p = 1/\sqrt{L \cdot C}$

$$Z_o = 62 \Omega \text{ and } Z_o = \sqrt{L/C}$$

$$\text{So } L = \frac{Z_o}{v_p} = 3.87 \times 10^{-7} \text{ H/m} \text{ and } C = \frac{1}{v_p Z_o} = 1.01 \times 10^{-10} \text{ F/m}$$

Conductor loss e:  eling

$R = 22.6 \Omega/\text{m}, L = 387 \text{ mH}$

$R_x = -3206, I_x = 20.64, M_x = 3206, P_x = 3.14$  •

$\alpha = 0.182 \text{ Np/m}$  or  $1.58 \text{ dB/m}$  •

Conversely, for low-loss transmission lines (i.e.,  $R \ll \omega L$ ) the attenuation can be accurately approximated using •

$$\alpha(\text{Np/m}) \approx \frac{R}{2 Z_0} = \frac{22.6 \Omega \text{ m}^{-1}}{2 (62 \Omega)} = 0.182 \text{ Np/m}$$

or  $1.58 \text{ dB/m}$

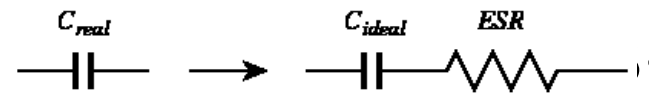
# Transmission line modeling <sup>Dielectric loss</sup>

For a dielectric with a non-zero conductivity ( $\sigma > 0$ ) (i.e., a non-infinite resistivity,  $\rho < \infty$ ), losses in the dielectric also attenuate the signal

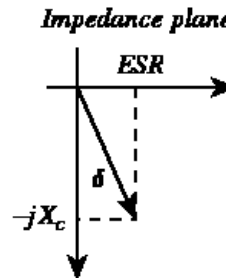
$\epsilon = \epsilon' + j\epsilon''$  permittivity becomes complex, •  
 with  $\epsilon'$  being the real part  
 $\epsilon''$  the imaginary part

$\epsilon'' = \sigma / \omega$  (conductivity of dielectric /  $2\pi f$ ) •

Sometimes specified as the *loss tangent*,  $\tan \delta$ , (the material characteristics table) •  
 $\tan \delta = \frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega \epsilon'}$



The term lo  
 displa



Don't confuse

ESR: equiv. series resistance •



# Transmission line modeling

From electromagnetic analysis we can relate  $\epsilon''$  to  $\alpha$

$$\alpha = \frac{2\pi}{\lambda_0} \left\{ \frac{\epsilon'}{2\epsilon_0} \left[ \sqrt{1 + \tan^2 \delta} - 1 \right] \right\}^{1/2}, \quad (\text{Neper / m})$$

- $\lambda_0$  = free-space wavelength =  $c/f$ ,  $c$  = speed of light
- $\epsilon_0$  = free-space permittivity =  $8.854 \times 10^{-12}$  F/m

- $\alpha_d$  (attenuation due to lossy dielectric) is frequency dependent ( $1/\lambda_0$ ) and it increases with frequency (heating of dielectric)

- Transmission line loss has two components,  $\alpha = \alpha_c + \alpha_d$
- Attenuation is present and it increases with frequency

- Can distort the signal
- reduces the signal level,  $V_o = V_s e^{-\alpha z}$
- reduces high-frequency components more than low-frequency components, increasing rise time

- Length is also a factor – for short distances (relative to  $\lambda$ )  $\alpha$  may be very small

# Transmission line modeling

Propagation velocity,  $v_p$  (or Delay) •

from circuit model,  $v_p = \frac{1}{\sqrt{LC}}$  (D = 1/ $v_p$ )

$$v_p = \frac{1}{\sqrt{\mu\epsilon}}$$

as long as L and C (or  $\mu$  and  $\epsilon$ ) are frequency independent, •  
 $v_p$  is frequency independent

# Transmission line modeling

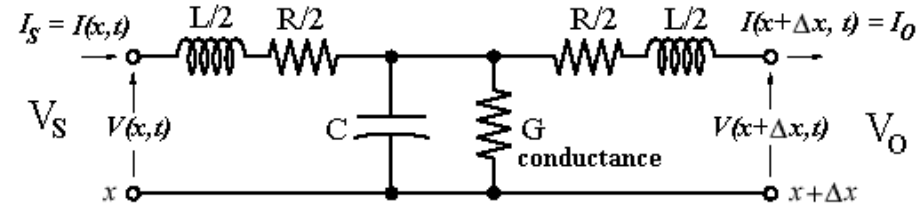
Characteristic impedance,  $Z_o$   
ratio of voltage to current,  $V/I = Z$

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$Z_o = \sqrt{\frac{R + j\omega L}{j\omega C}}$$

$$Z_o \cong \sqrt{\frac{R}{j\omega C}}$$

$$Z_o \cong \sqrt{\frac{L}{C}}$$



typically  $G$  (conductance) is very small  $\approx 0$ , so •

At low frequencies,  $R \gg \omega L$  when  $f \ll R/(2\pi L)$  •  
the transmission line behaves as an R-C circuit

$Z_o$  is complex

$Z_o$  is frequency dependent

At high frequencies,  $\omega L \gg R$  when  $f \gg R/(2\pi L)$  •

$Z_o$  is real

$Z_o$  is frequency independent

# Characteristic impedance example • transmission line modeling

$R = 2.6 \text{ m}\Omega / \text{inch} = 0.1 \text{ }\Omega / \text{m}$  consider a transmission line with

$L = 6.35 \text{ nH} / \text{inch} = 250 \text{ nH} / \text{m}$

$C = 2.54 \text{ pF} / \text{inch} = 100 \text{ pF} / \text{m}$

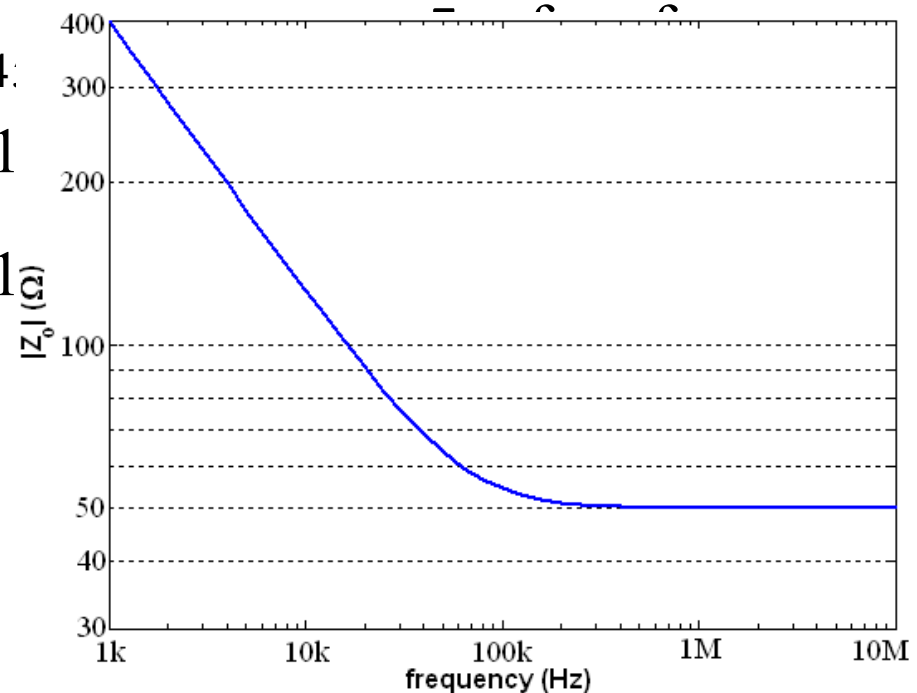
From these parameters we know, •

$v_p = 2 \times 10^8 \text{ m/s} = 0.67 c \rightarrow \epsilon_r = 2.25$

$f_1 = R/(2\pi L) = 63.7 \text{ kHz}$

$$Z_o \cong \sqrt{\frac{R}{j\omega C}} \cong \frac{12.6 \text{ k}\Omega}{\sqrt{f}} \angle -45^\circ$$

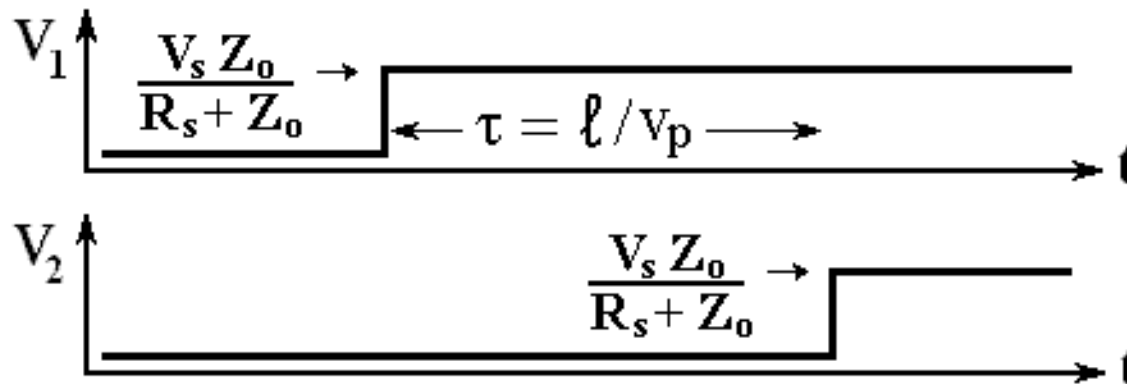
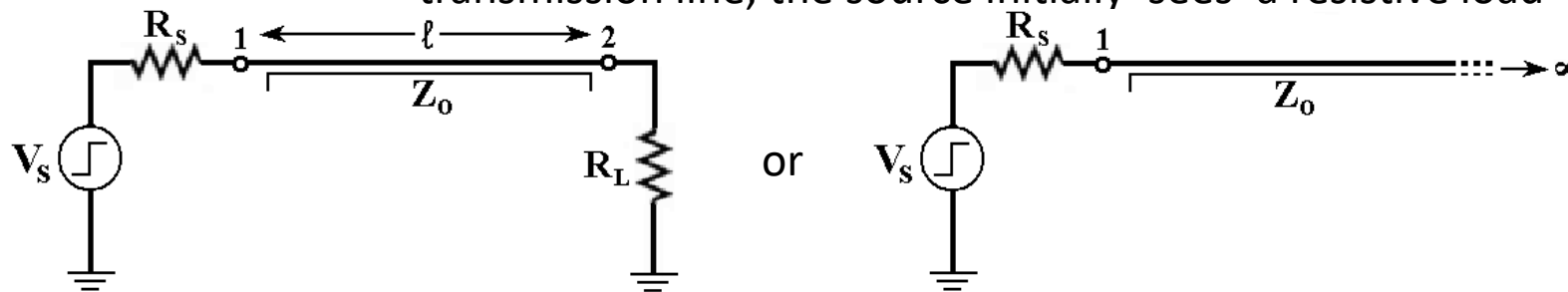
$$|Z_o| \approx 1$$



# Traveling waves • Transmission line modeling

For frequencies above  $f_1$  (and frequency components above  $f_1$ ),  
 $Z_0$  is real and frequency independent

For a transmission line with a matched load ( $R_L = Z_0$ ), when 'looking' into a transmission line, the source initially 'sees' a resistive load



# Transmission line modeling

Consider the case where the 5-V step voltage generator with a 30- $\Omega$  source resistance drives a 50- $\Omega$  transmission line with a matched impedance termination.

$$V_s = 5 \text{ V}, R_s = 30 \text{ } \Omega, Z_o = 50 \text{ } \Omega, R_L = 50 \text{ } \Omega \quad \bullet$$

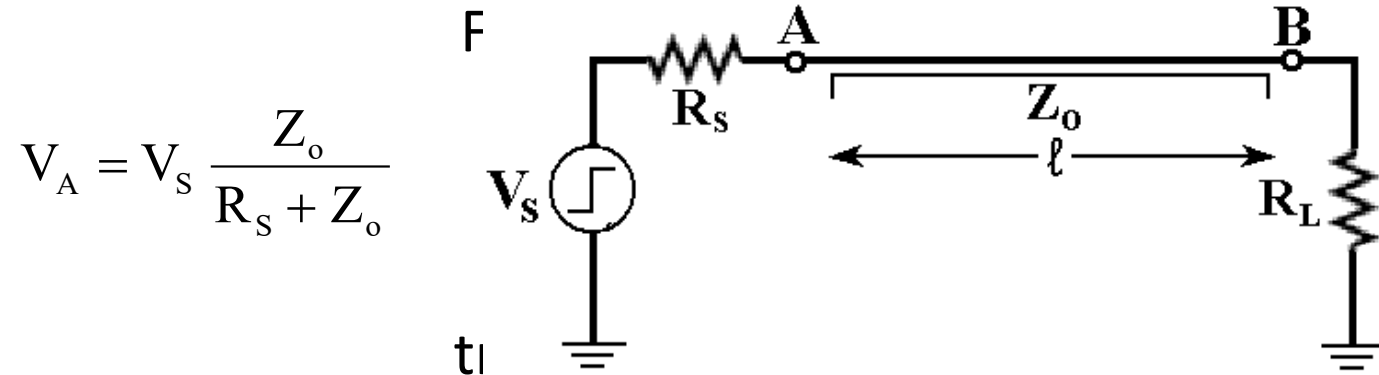
The wave propagating down the transmission line has a voltage of •

The wave propagating down the transmission line has a current of •

$$\frac{3.13 \text{ V}}{50 \text{ } \Omega} = 63 \text{ mA}$$

# Transmission line modeling

Now consider the more general case with impedance mismatches



$$V_A = V_s \frac{Z_o}{R_s + Z_o}$$

at B at time  $t = \ell/v_p$  where the impedance mismatch causes a reflected wave with reflection coefficient,  $\rho_L$

→ wave voltage =  $V_A \rho_L$  •

$$\rho_L = \frac{R_L - Z_o}{R_L + Z_o}$$

Special cases •

- if  $R_L = Z_o$  (matched impedance), then  $\rho_L = 0$  •
- if  $R_L = 0$  (short circuit), then  $\rho_L = -1$  •
- if  $R_L = \infty$  (open circuit), then  $\rho_L = +1$  •

# Transmission line modeling

The reflected signal then travels back down the transmission line and arrives at A at time  $t = 2\ell/v_p$  where another impedance mismatch causes a reflected wave with reflection coefficient,  $\rho_S$

$$\rho_S = \frac{R_S - Z_o}{R_S + Z_o}$$

→ wave voltage  $V_A \rho_L \rho_S$

This reflected signal again travels down the transmission line ...

For complex loads, the reflection coefficient is complex

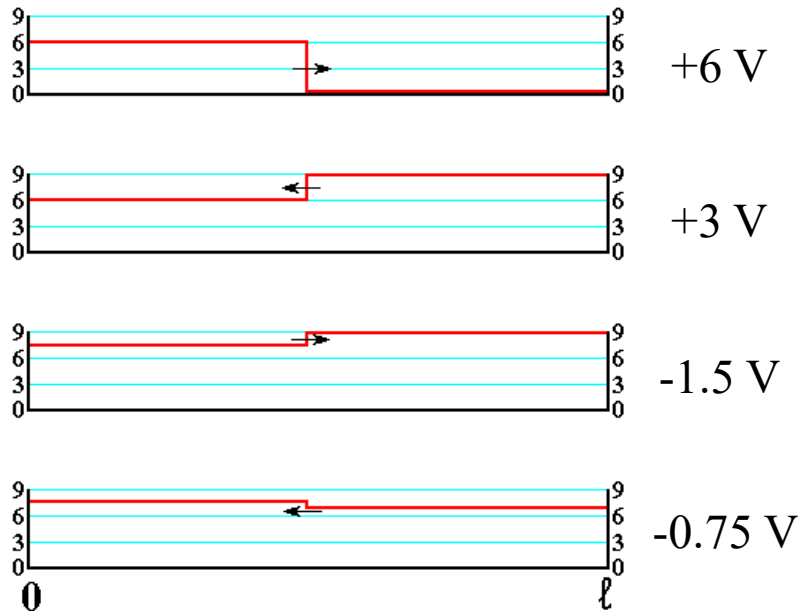
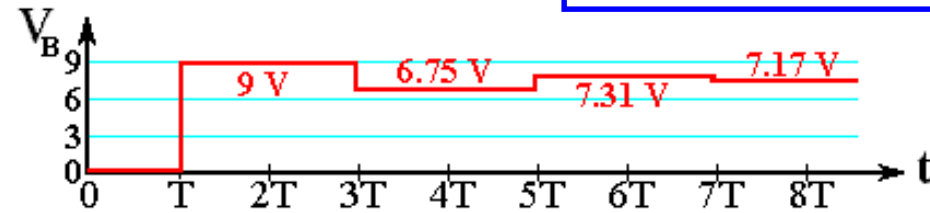
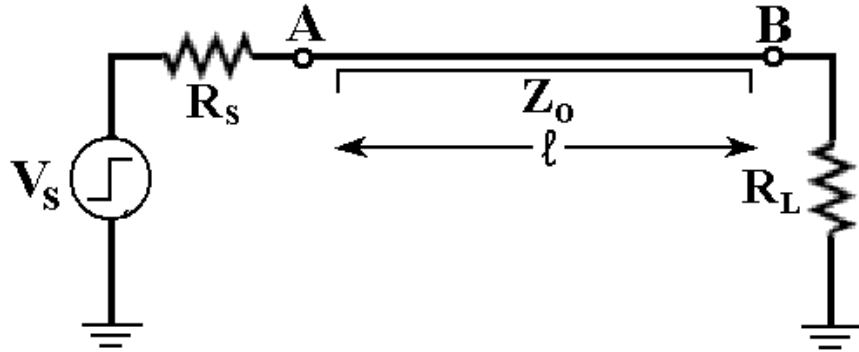
$$\rho = \frac{Z_L - Z_o}{Z_L + Z_o} \quad \text{or} \quad \frac{Z_L}{Z_o} = \frac{1 + \rho}{1 - \rho} \quad \text{In general}$$



# Transmission line modeling

Example,  $Z_0 = 60 \Omega$ ,  $Z_S = 20 \Omega$ ,  $Z_L = 180 \Omega$ ,  $V_S = 8 \text{ V}$ ,  $V_A = 6 \text{ V}$ ,  $R_L = 180 \Omega$   
 $\rightarrow \rho_L = +1/2$ ,  $\rho_S = -1/2$ ,  $V_A = 6 \text{ V}$ ,  $I = 33.3 \text{ mA}$

Traveling waves:  
 $V_A = 6 \text{ V}$ ,  $I = 33.3 \text{ mA}$   
 Excess current 66.7 mA



Multiple reflections result when  $R_S \neq Z_0$ ,  $R_L \neq Z_0$

Ringing results when  $\rho_L$  and  $\rho_S$  are of opposite polarity

Ideally we'd like no reflections,  $\rho = 0$   
 Realistically, some reflections exist

How much  $\rho$  can we tolerate?  
 Answer depends on noise margin

# Reflections and noise margin

## Transmission line modeling

Consider the ECL noise margin

$$\frac{V_{OH}(min) - V_{IH}(max)}{V_{OH}(max) - V_{OL}(min)} \times 100\% = \frac{-1025 - (-870)}{-870 - (-1830)} \times 100\% = 16\%$$

Terminal voltage is the sum of incident wave and reflected wave  $v_i(1+\rho)$  •  
 noise margin  $\sim \rho_{max} \rightarrow \rho_{max}$  for ECL  $\sim 15\%$

$$\frac{R_L}{Z_o} = \frac{1+\rho}{1-\rho} \quad , \text{ evaluate for } \rho = \pm 0.15 \quad \bullet$$

$$\frac{1.15}{0.85} > \frac{R_L}{Z_o} > \frac{0.85}{1.15} \rightarrow 1.35 > \frac{R_L}{Z_o} > 0.74$$

$$\text{or } 1.35 Z_o > R_L > 0.74 Z_o$$

For  $Z_o = 50 \Omega$ ,  $67.5 \Omega > R_L > 37 \Omega$  however  $Z_o$  variations also contribute to  $\rho$  •  
 if  $R_T = 50 \Omega$  ( $\pm 10\%$ ), then  $Z_o = 50 \Omega$  (+22% or -19%)  
 if  $R_T = 50 \Omega$  ( $\pm 5\%$ ), then  $Z_o = 50 \Omega$  (+28% or -23%)

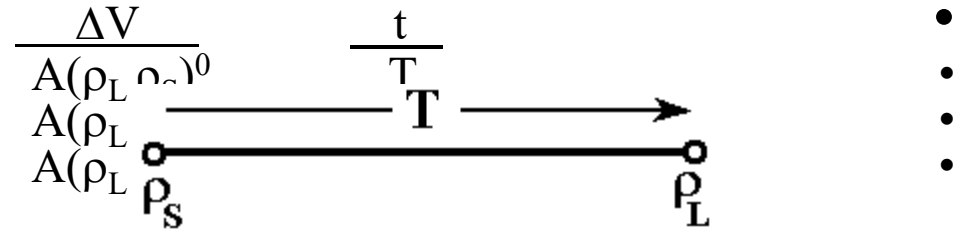
# Reflections Transmission line modeling

How long does it take for transients to settle?

Settling criterion : mag. of change rel. to original wave < some value, a

$\Delta V$ : magnitude of traveling wave, A: magnitude of original wave  
 $|\Delta V/A| < a$

The signal amplitude just before the termination vs. time



From this pattern we know

for a given a,  $\rho_L$ ,  $\rho_S$ , solve for  $t_a$ ,  
 $\frac{\Delta V}{A} = \left( \frac{|\rho_L \rho_S|}{2} \right)^{\frac{t}{T}} < a$

Example,  $\rho_S = 0.8$ ,  $\rho_L = 0.15$ ,  $a = 10\%$ ,  $T = 10 \text{ ns} \rightarrow t_a = 3.17 T$

$$t_a = T \left[ 2 \frac{\ln a}{\ln (|\rho_L \rho_S|)} + 1 \right] \sim 30 \text{ ns}$$

To reduce  $t_a$ , must reduce either  $\rho_S$ ,  $\rho_L$ , or T (or  $\ell$ )

# Transm

$$Z_o = \frac{60}{\sqrt{\epsilon_r}} \ln\left(\frac{b}{a}\right), \Omega$$

$$v_p = c/\sqrt{\epsilon_r}, \text{ m/s}$$

frequency independent

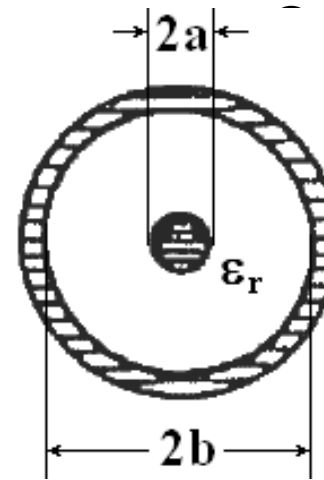
$$\alpha = \alpha_c + \alpha_d$$

$$\alpha_c = \frac{R_s}{4\pi Z_o} \left(\frac{1}{b} + \frac{1}{a}\right), \text{ Neper/m}$$

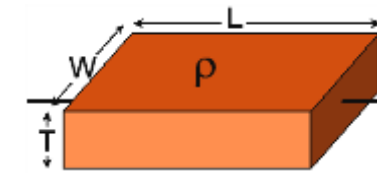
frequency dependent

$$\alpha_d = \frac{\pi \sqrt{\epsilon_r} \tan \delta}{\lambda_o}, \text{ Neper/m}$$

$$\lambda_o = c/f, \text{ free-space wavelength}$$



# axial cable e structures



$R_s$  is sheet resistivity ( $\Omega$  per square or  $\Omega/\square$ )

$R_s = \rho/T$ ,  $\rho$  = resistivity,  $T$  = thickness

sometimes written as  $R_{\square}$

real units are  $\Omega$ , "square" is unitless

$$R = R_s L/W$$

Assuming the skin depth determines  $T$   
attenuation in dB/m is  $8.686 (\alpha_c + \alpha_d)$

$$T = \sqrt{\frac{2\rho}{\omega\mu_o}} = \sqrt{\frac{\rho}{\pi f \mu_o}}$$

$$\rightarrow R_s = \rho \cdot \sqrt{\frac{\pi f \mu_o}{\rho}} = \sqrt{\pi f \mu_o \rho}$$

# Transmission line structures

Coaxial cable  
Other parameters of interest

Useful frequencies of operation

$$f_1 = \text{DC}$$

Transverse electromagnetic (TEM) mode is the desired propagation mode

Above the cutoff frequency,  $f_c$ , another mode can be supported,  $TE_{01}$

This determines the upper frequency limit for operation as transmission

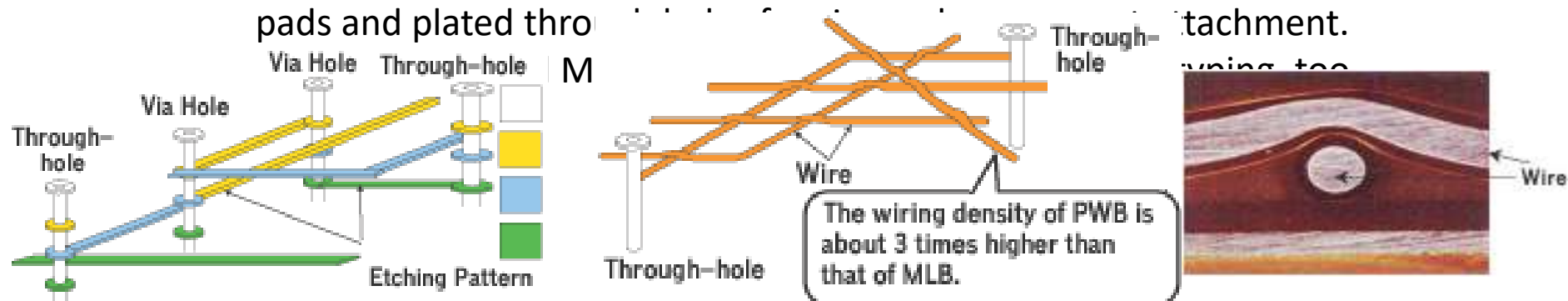
line characteristics are different for the new mode ( $Z_o, v_p, \alpha$ )

$$f_2 = f_c = \frac{c}{\pi(a+b)}$$

Coaxial lines can be used for point-to-point wiring

In PCB, the coax is laid out and held in place with epoxy (no crosstalk)

Lines are cut, the center conductor exposed, and metal deposited to provide pads and plated through-holes for attachment.



# Transmission line structures

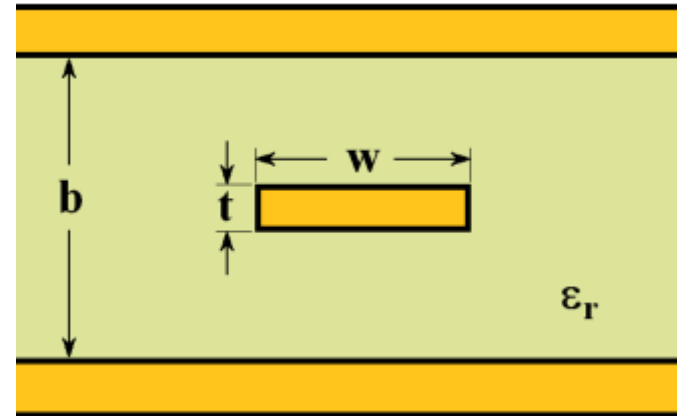
Stripline •

$Z_o = f(t, \epsilon_r, w, b)$  complex relationship

$$v_p = \frac{c}{\sqrt{\epsilon_r}}$$

$$\alpha_c = f(\rho, \epsilon_r, Z_o, b, t, w, f)$$

$$\alpha_d = f(\epsilon_r, \tan \delta, f)$$



$I_1 = DC, I_2 = I(\epsilon_r, D, W, t)$  •

Expressions are provided in supplemental information on course website

Simpler expressions for  $Z_o$  available

Found in other sources •

e.g., eqn. 4.92 on pg 188 in text, Appendix C of text pgs 436-439

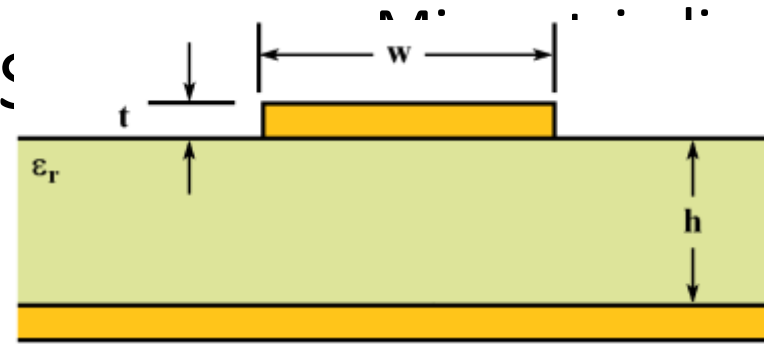
$$Z_o = \frac{60}{\sqrt{\epsilon_r}} \ln \left( \frac{1.9b}{0.8w + t} \right), \text{ Less accurate } \bullet$$

Only applicable for certain range of geometries (e.g.  $w/b$  ratios) •  
 For  $w/b < 0.95$  and  $t/b < 0.25$

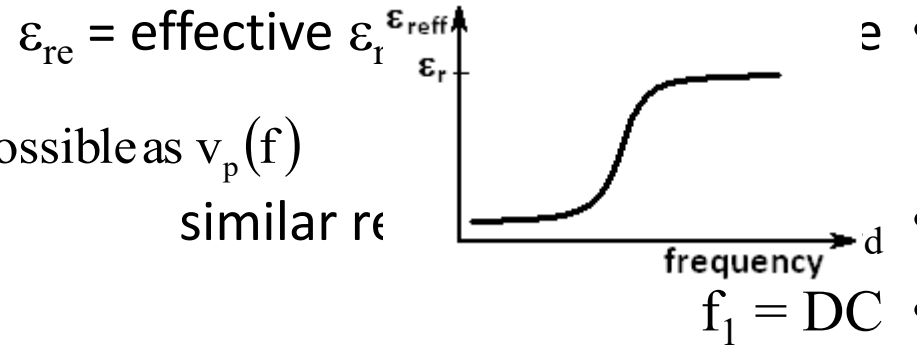
# Transmission Line Structures

$$Z_o = f(t, \epsilon_r, h, w)$$

$$v_p = \frac{c}{\sqrt{\epsilon_{re}}}$$



$$\epsilon_{re} = f(\epsilon_r, h, w, t, f) \rightarrow \text{dispersion possible as } v_p(f)$$



Expressions are provided in supplemental information on course website

Simpler expressions for  $Z_o$  available

Found in other sources •

e.g., eqn 4.90 on pg 187 in text, Appendix C of text pgs 430-435

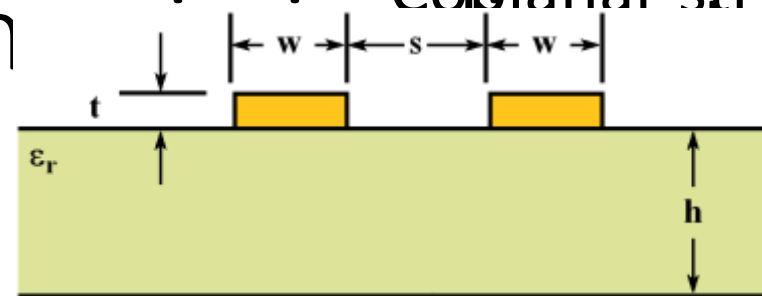
$$Z_o = \frac{87}{\sqrt{\epsilon_r + 1.41}} \ln \left( \frac{5.98 h}{0.8 w + t} \right), \quad \Omega \quad \text{for } 0.1 < w/h < 2.0 \quad \text{and } 1 < \epsilon_r < 15$$

Less accurate •  
Only applicable for certain range of geometries (e.g., w/h ratios) •

# Transverse Co-planar strips structures

$$Z_o = \frac{120\pi}{\sqrt{\epsilon_{re}}} K(k)/K'(k)$$

$$\frac{K(k)}{K'(k)} = \begin{cases} \left[ \frac{1}{\pi} \ln \left( 2 \frac{1 + \sqrt{k'}}{1 - \sqrt{k'}} \right) \right]^{-1} & \text{for } 0 \leq k < 0.7 \\ \frac{1}{\pi} \ln \left( 2 \frac{1 + \sqrt{k}}{1 - \sqrt{k}} \right) & \text{for } 0.7 \leq k \leq 1 \end{cases}$$



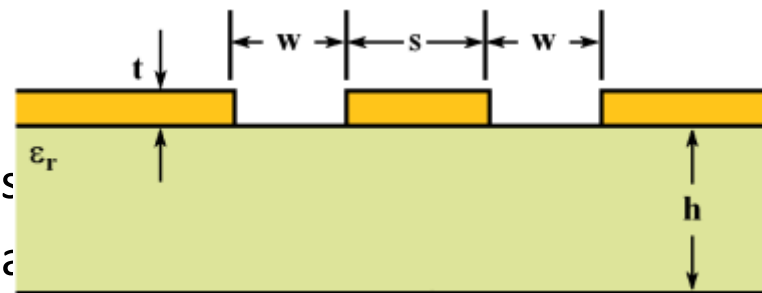
where  $k = \frac{s}{s + 2w}$  and  $k' = \sqrt{1 - k^2}$

$$\epsilon_{re} = \frac{\epsilon_r + 1}{2} \left\{ \tanh[0.775 \ln(h/w) + 1.75] + k w/h [0.04 - 0.7k + 0.6k^2 - 0.1\epsilon_r(0.5 + k)] \right\}$$

## Co-planar waveguide

$$Z_o = \frac{30\pi}{\sqrt{\epsilon_{re}}} K'(k)/K(k)$$

Expressions for  $Z_o$ ,  $\alpha_c$ ,  $\alpha_d$ ,  $\beta$



information on course website



# Effects of manufacturing variations on electrical transmission line structures

Specific values for transmission line parameters from design equations

$$t, \epsilon_r, w, b \text{ or } h \rightarrow Z_o, v_p, \alpha$$

However manufacturing processes are not perfect fabrication errors introduced

Example – the trace width is specified to be 10 mils due to process variations the fabricated trace width is  $w = 10 \text{ mils} \pm 2.5 \text{ mils}$  ( $3\sigma$  variation  $\rightarrow$  99.5% of traces lie within these limits)

The board manufacturer specifies  $\epsilon_r = 4 \pm 0.1$  ( $3\sigma$ ) and  $h = 10 \text{ mil} \pm 0.2 \text{ mil}$  ( $3\sigma$ )

These uncertainties contribute to electrical performance variations

$Z_o \pm \Delta Z_o, v_p \pm \Delta v_p$   
These effects must be considered

So given  $\Delta t, \Delta \epsilon_r, \Delta w, \Delta h$ , how is  $\Delta Z_o$  found ?

# Effects of manufacturing variations on electrical transmission line structures performance

Various techniques available for finding  $\sigma_{Z_0}$  due to  $\sigma_t, \sigma_w, \sigma_{\epsilon_r}$  •

Monte-Carlo simulation •

Let each parameter independently vary randomly and find the  $Z_0$

Repeat a large number of trials

Find the mean and standard deviation of the  $Z_0$  results

Find the sensitivity of  $Z_0$  to variations in each parameter ( $t, \epsilon_r, w, h$ ) •  
 ( $Z(\text{nom}), t(\text{nom}), \epsilon_r(\text{nom}), \dots$ ) to determine  $Z_0 = Z_0(\text{nom}) \pm 3\sigma_{Z_0}$

For example find  $\partial Z_0 / \partial t$  to find the sensitivity to thickness variations about the nominal values

For small perturbations ( $\sigma$ ) treat the relationship  $Z_0(t, \epsilon_r, w, h)$  as linear where  $\partial Z_0 / \partial t$  is found through numerical differentiation

$$\sigma_{Z_0|t} = \frac{\partial Z_0}{\partial t} \cdot \sigma_t$$

$$\frac{\partial Z_0}{\partial t} = \frac{Z_0(t + \epsilon_r, w, h) - Z_0(t - \epsilon_r, w, h)}{\Delta t}$$

with respect to each parameter

$$\sigma_{Z_0|w} = \frac{\partial Z_0}{\partial w} \cdot \sigma_w$$

$$\sigma_{Z_0|\epsilon_r} = \frac{\partial Z_0}{\partial \epsilon_r} \cdot \sigma_{\epsilon_r}$$

$$\sigma_{Z_0|h} = \frac{\partial Z_0}{\partial h} \cdot \sigma_h$$

combine the various  $\sigma_{Z_0}$  terms in a root-sum-square (RSS) fashion

$$\sigma_{Z_0} = \sqrt{\sigma_{Z_0|t}^2 + \sigma_{Z_0|\epsilon_r}^2 + \sigma_{Z_0|w}^2 + \sigma_{Z_0|h}^2}$$

# Transmission line summary

- Overview of transmission lines
- Requirements and characteristics of interest
- Types of transmission lines
- Parameters that contribute to characteristics of interest

## Transmission line modeling •

Circuit model •

Relating  $R$ ,  $L$ ,  $C$ ,  $G$  to  $Z_0$ ,  $v_p$ ,  $\alpha$  •

Conductive loss factors •

Resistivity, skin effect, proximity effect •

Dielectric loss factors •

Conduction loss, polarization loss → loss tangent •

Wave propagation and reflections •

Reflections at source and load ends •

Voltage on transmission line vs. time •

## Design equations for transmission line structures •

Coaxial cable, stripline, microstrip, coplanar strips and waveguide •

Analysis of impact of manufacturing variations on  $Z_0$  •

# Why are 50-Ω transmission lines used? • Transmission line bonus

It is the result of a compromise between minimum insertion loss and maximum power handling capability

$Z_0 \sim 77 \Omega$  for  $\epsilon_r = 1$  (air) Insertion loss is minimum at:

$Z_0 \sim 64 \Omega$  for  $\epsilon_r = 1.43$  (PTFE foam)

$Z_0 \sim 50 \Omega$  for  $\epsilon_r = 2.2$  (solid PTFE)

Power handling is maximum at  $Z_0 = 30 \Omega$

Voltage handling is maximum at  $Z_0 \sim 60 \Omega$

•  
For printed-circuit board transmission lines –

Near-field EMI  $\propto$  height above plane,  $h$

Crosstalk  $\propto h^2$

Impedance  $\propto h$

Higher impedance lines are difficult to fabricate (especially for small  $h$ )